CHAPTER

12

BERNOULLI AND ENERGY EQUATIONS

This chapter deals with two equations commonly used in fluid mechanics: the Bernoulli equation and the energy equation. The *Bernoulli equation* is concerned with the conservation of kinetic, potential, and flow energies of a fluid stream, and their conversion to each other in regions of flow where net viscous forces are negligible, and where other restrictive conditions apply. The *energy equation* is a statement of the conservation of energy principle and is applicable under all conditions. In fluid mechanics, it is found to be convenient to separate *mechanical energy* from *thermal energy* and to consider the conversion of mechanical energy to thermal energy as a result of frictional effects as *mechanical energy loss.* Then the energy equation is usually expressed as the *conservation of mechanical energy.*

We start this chapter with a discussion of various forms of mechanical energy and the efficiency of mechanical work devices such as pumps and turbines. Then we derive the Bernoulli equation by applying Newton's second law to a fluid element along a streamline and demonstrate its use in a variety of applications. We continue with the development of the energy equation in a form suitable for use in fluid mechanics and introduce the concept of *head loss.* Finally, we apply the energy equation to various engineering systems.

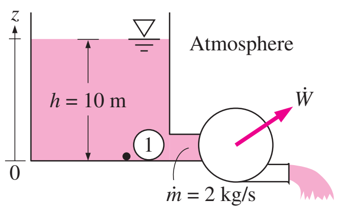
CONTENTS

12–1 ▪ MECHANICAL ENERGY AND EFFICIENCY

Most fluid systems are designed to transport a fluid from one location to another at a specified flow rate, velocity, and elevation difference, and the system may generate mechanical work in a turbine or it may consume mechanical work in a pump or fan during this process. These systems do not involve the conversion of nuclear, chemical, or thermal energy to mechanical energy. Also, they do not involve any heat transfer in any significant amount, and they operate essentially at constant temperature. Such systems can be analyzed conveniently by considering the *mechanical forms of energy* only and the frictional effects that cause the mechanical energy to be lost (i.e., to be converted to thermal energy that usually cannot be used for any useful purpose).

The **mechanical energy** can be defined as *the form of energy that can be converted to mechanical work completely and directly by a mechanical device such as an ideal turbine.* Kinetic and potential energies are the familiar forms of mechanical energy. Thermal energy is not mechanical energy, however, since it cannot be converted to work directly and completely (the second law of thermodynamics).

A pump transfers mechanical energy to a fluid by raising its pressure, and a turbine extracts mechanical energy from a fluid by dropping its pressure. Therefore, the pressure of a flowing fluid is also associated with its mechanical energy. In fact, the pressure unit Pa is equivalent to Pa $=$ N/m^2 $=$ N $\cdot$ m/m^3 $=$ J/m^3, and the product $P\upsilon$ or its equivalent P/ρ has the unit J/kg, which is energy unit per unit mass. Note that pressure itself is not a form of energy. But a pressure force acting on a fluid through a distance produces work, called *flow work,* in the amount of P/ρ per unit mass. Flow work is expressed in terms of fluid properties, and it is found convenient to view it as part of the energy of a flowing fluid, and call it *flow energy.* Therefore, the mechanical energy of a flowing fluid can be expressed on a unit mass basis as (Fig. 12–1).



$$\dot{W}_{max} = \dot{m}\frac{P_1 - P_{atm}}{\rho} = \dot{m}\frac{\rho g h}{\rho} = \dot{m}gh$$
$$= (2 \text{ kg/s})(9.81 \text{ m/s}^2)(10 \text{ m})$$
$$= 196 \text{ W}$$

FIGURE 12–1
In the absence of any changes in flow velocity and elevation, the power produced by a hydraulic turbine is proportional to the pressure drop of water across the turbine.

$$e_{mech} = \frac{P}{\rho} + \frac{\mathcal{V}^2}{2} + gz$$

where P/ρ is the *flow energy*, $\mathcal{V}^2/2$ is the *kinetic energy*, and gz is the *potential energy* of the fluid per unit mass. Then the mechanical energy change of a fluid during incompressible flow becomes

$$\Delta e_{mech} = \frac{P_2 - P_1}{\rho} + \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} + g(z_2 - z_1) \quad \text{(kJ/kg)} \qquad \textbf{(12–1)}$$

Therefore, the mechanical energy of a fluid does not change during flow if its pressure, density, velocity, and elevation remain constant. In the absence of any losses, the mechanical energy change represents the mechanical work supplied to the fluid (if $\Delta e_{mech} > 0$) or extracted from the fluid (if $\Delta e_{mech} < 0$).

Consider a container of height h filled with water, as shown in Fig. 12–2, with reference level selected at the bottom surface. The gage pressure and the potential energy per unit mass are $P_A = 0$ and $pe_A = gh$ at point A at the free surface, and $P_B = \rho gh$ and $pe_B = 0$ at point B at the bottom of the container. An ideal hydraulic turbine would produce the same work per unit mass $w_{turbine} = gh$ whether it receives water (or any other fluid with constant density) from the top or from the bottom of the container. Note that we are

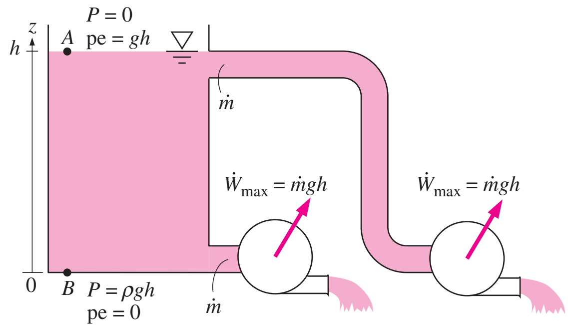


FIGURE 12–2
The mechanical energy of water at the bottom of a lake is equal to the mechanical energy at any depth including the free surface of the lake.

also assuming ideal flow (no irreversible losses) through the pipe leading from the tank to the turbine. Therefore, the total mechanical energy of water at the bottom is equivalent to that at the top.

The transfer of mechanical energy is usually accomplished by a rotating shaft, and thus mechanical work is often referred to as *shaft work.* A pump or a fan receives shaft work (usually from an electric motor) and transfers it to the fluid as mechanical energy (less frictional losses). A turbine, on the other hand, converts the mechanical energy of a fluid to shaft work. In the absence of any irreversibilities such as friction, mechanical energy can be converted entirely from one mechanical form to another, and the **mechanical efficiency** of a device or process can be defined as (Fig. 12–3).

$$\eta_{\text{mech}} = \frac{\text{Mechanical energy output}}{\text{Mechanical energy input}} = \frac{E_{\text{mech, out}}}{E_{\text{mech, in}}} = 1 - \frac{E_{\text{mech, loss}}}{E_{\text{mech, in}}} \qquad \text{(12–2)}$$

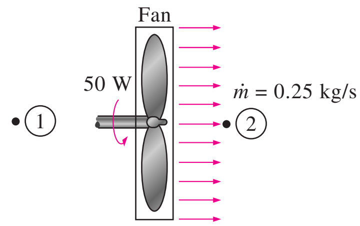
A conversion efficiency of less than 100 percent indicates that conversion is less than perfect and some losses have occurred during conversion. A mechanical efficiency of 97 percent indicates that 3 percent of the mechanical energy input is converted to thermal energy as a result of frictional heating, and this will manifest itself as a slight rise in the temperature of the fluid.

In fluid systems, we are usually interested in increasing the pressure, velocity, and/or elevation of a fluid. This is done by *supplying mechanical energy* to the fluid by a pump, a fan, or a compressor (we will refer to all of them as pumps). Or we are interested in the reverse process of *extracting mechanical energy* from a fluid by a turbine, and producing mechanical power in the form of a rotating shaft that can drive a generator or any other rotary device. The degree of perfection of the conversion process between the mechanical work supplied or extracted and the mechanical energy of the fluid is expressed by the **pump efficiency** and **turbine efficiency,** defined as

$$\eta_{\text{pump}} = \frac{\text{Mechanical energy increase of the fluid}}{\text{Mechanical energy input}} = \frac{\dot{W}_{\text{pump, u}}}{\dot{W}_{\text{pump}}} \qquad \text{(12–3)}$$

where $\dot{W}_{\text{pump, u}} = \Delta\dot{E}_{\text{mech, fluid}} = \dot{E}_{\text{mech, out}} - \dot{E}_{\text{mech, in}}$ is the rate of increase in the mechanical energy of the fluid, which is equivalent to the **useful pumping power** supplied to the fluid, and

$$\eta_{\text{turbine}} = \frac{\text{Mechanical engery output}}{\text{Mechanical energy decrease of the fluid}} = \frac{\dot{W}_{\text{shaft, out}}}{|\Delta\dot{E}_{\text{mech, fluid}}|} = \frac{\dot{W}_{\text{turbine}}}{\dot{W}_{\text{turbine, e}}} \qquad \text{(12–4)}$$



$V_1 = 0,\ V_2 = 12$ m/s
$z_1 = z_2$
$P_1 = P_2$

$$\eta_{\text{mech, fan}} = \frac{\Delta\dot{E}_{\text{mech, fluid}}}{\dot{W}_{\text{shaft, in}}} = \frac{\dot{m}V_2^2/2}{\dot{W}_{\text{shaft, in}}} = \frac{(0.25\ \text{kg/s})(12\ \text{m/s})^2/2}{50\ \text{W}} = 0.72$$

FIGURE 12–3
The mechanical efficiency of a fan is the ratio of the kinetic energy of air at the fan exit to the mechanical power input.

where $|\Delta \dot{E}_{\text{mech, fluid}}| = \dot{E}_{\text{mech, in}} - \dot{E}_{\text{mech, out}}$ is the rate of decrease in the mechanical energy of the fluid, which is equivalent to the mechanical power extracted from the fluid by the turbine $\dot{W}_{\text{turbine, e}}$, and we used the absolute value sign to avoid negative values for efficiencies. A pump or turbine efficiency of 100 percent indicates perfect conversion between the shaft work and the mechanical energy of the fluid, and this value can be approached (but never attained) as the frictional effects are minimized.

The mechanical efficiency should not be confused with the **motor efficiency** and the **generator efficiency,** which are defined as

Motor:

$$\eta_{\text{motor}} = \frac{\text{Mechanical power output}}{\text{Electrical power input}} = \frac{\dot{W}_{\text{shaft, out}}}{\dot{W}_{\text{elect, in}}} \tag{12–5}$$

and

Generator:

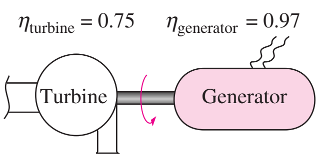
$$\eta_{\text{generator}} = \frac{\text{Electrical power output}}{\text{Mechanical power input}} = \frac{\dot{W}_{\text{elect, out}}}{\dot{W}_{\text{shaft, in}}} \tag{12–6}$$

A pump is usually packaged together with its motor, and a hydraulic turbine with its generator. Therefore, we are usually interested in the **combined** or **overall efficiency** of pump/motor and turbine/generator combinations (Fig. 12–4), which are defined as

$$\eta_{\text{pump-motor}} = \eta_{\text{pump}}\,\eta_{\text{motor}} = \frac{\dot{W}_{\text{pump, u}}}{\dot{W}_{\text{elect, in}}} = \frac{\Delta \dot{E}_{\text{mech, fluid}}}{\dot{W}_{\text{elect, in}}} \tag{12–7}$$

and

$$\eta_{\text{turbine-gen}} = \eta_{\text{turbine}}\,\eta_{\text{generator}} = \frac{\dot{W}_{\text{elect, out}}}{\dot{W}_{\text{turbine, e}}} = \frac{\dot{W}_{\text{elect, out}}}{|\Delta \dot{E}_{\text{mech, fluid}}|} \tag{12–8}$$



$$\eta_{\text{turbine-gen}} = \eta_{\text{turbine}}\,\eta_{\text{generator}} = 0.75 \times 0.97 = 0.73$$

FIGURE 12–4
The overall efficiency of a turbine-generator is the product of the efficiency of the turbine and the efficiency of the generator, and represents the fraction of the mechanical energy of the fluid converted to electrical energy.

All the efficiencies just defined range between 0 and 100 percent. The lower limit of 0 percent corresponds to the conversion of the entire mechanical or electrical energy input to thermal energy, and the device in this case functions like a resistance heater. The upper limit of 100 percent corresponds to the case of perfect conversion with no friction or other irreversibilities, and thus no conversion of mechanical or electrical energy to thermal energy.

EXAMPLE 12–1 Performance of a Hydraulic Turbine-Generator

The water in a large lake is to be used to generate electricity by installing a hydraulic turbine-generator at a location where the depth of the water is 50 m (Fig. 12–5). Water is to be supplied at a rate of 5000 kg/s. If the electric power generated is measured to be 1862 kW and the generator efficiency is 95 percent, determine (*a*) the overall efficiency of the turbine-generator, (*b*) the mechanical efficiency of the turbine, and (*c*) the shaft power supplied by the turbine to the generator.

SOLUTION A hydraulic turbine-generator is to generate electricity from the water of a lake. The overall efficiency, the turbine efficiency, and the shaft power are to be determined.

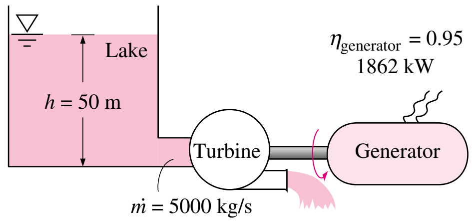


FIGURE 12–5
Schematic for Example 12–1.

Assumptions **1** The elevation of the lake remains constant. **2** The mechanical energy of water at the turbine exit is negligible.

Properties The density of water can be taken to be $\rho = 1000\ \text{kg/m}^3$.

Analysis (*a*) We take the bottom of the lake as the reference level for convenience. Then kinetic and potential energies of water are zero, and the change in its mechanical energy per unit mass becomes

$$e_{\text{mech, in}} - e_{\text{mech, out}} = \frac{P}{\rho} - 0 = gh = (9.81\ \text{m/s}^2)(50\ \text{m})\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = 0.491\ \text{kJ/kg}$$

Then the rate at which mechanical energy is supplied to the turbine by the fluid and the overall efficiency become

$$|\Delta \dot{E}_{\text{mech, fluid}}| = \dot{m}(e_{\text{mech, in}} - e_{\text{mech, out}}) = (5000\ \text{kg/s})(0.491\ \text{kJ/kg}) = 2455\ \text{kW}$$

$$\eta_{\text{overall}} = \eta_{\text{turbine-gen}} = \frac{\dot{W}_{\text{elect, out}}}{|\Delta \dot{E}_{\text{mech, fluid}}|} = \frac{1862\ \text{kW}}{2455\ \text{kW}} = \mathbf{0.76}$$

(*b*) Knowing the overall and generator efficiencies, the mechanical efficiency of the turbine is determined from

$$\eta_{\text{turbine-gen}} = \eta_{\text{turbine}}\,\eta_{\text{generator}} \quad \rightarrow \quad \eta_{\text{turbine}} = \frac{\eta_{\text{turbine-gen}}}{\eta_{\text{generator}}} = \frac{0.76}{0.95} = \mathbf{0.80}$$

(*c*) The shaft power output is determined from the definition of mechanical efficiency,

$$\dot{W}_{\text{shaft, out}} = \eta_{\text{turbine}}\,|\Delta \dot{E}_{\text{mech, fluid}}| = (0.80)(2455\ \text{kW}) = \mathbf{1964\ kW}$$

Discussion Note that the lake supplies 2455 kW of mechanical energy to the turbine, which converts 1964 kW of it to shaft work that drives the generator, which generates 1862 kW of electric power. There are irreversible losses through each component.

EXAMPLE 12–2 Conservation of Energy for an Oscillating Steel Ball

The motion of a steel ball in a hemispherical bowl of radius *h* shown in Fig. 12–6 is to be analyzed. The ball is initially held at the highest location at point *A*, and then it is released. Obtain relations for the conservation of energy of the ball for the cases of frictionless and actual motions.

SOLUTION A steel ball is released in a bowl. Relations for the energy balance are to be obtained.

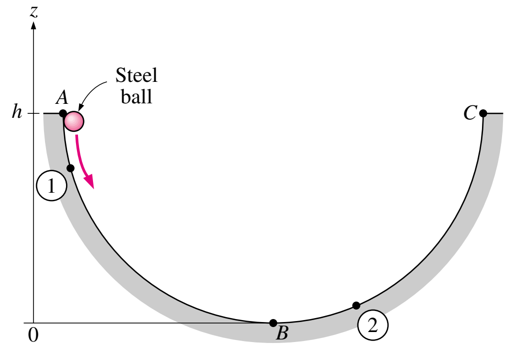


FIGURE 12–6
Schematic for Example 12–2.

Assumptions The motion is frictionless, and thus friction between the ball, the bowl, and the air is negligible.

Analysis When the ball is released, it accelerates under the influence of gravity, reach a maximum velocity (and minimum elevation) at point *B* at the bottom of the bowl, and move up toward point *C* on the opposite side. In the ideal case of frictionless motion, the ball will oscillate between points *A* and *C*. The actual motion involves the conversion of the kinetic and potential energies of the ball to each other, together with overcoming resistance to motion due to friction (doing frictional work). The general energy balance for any system undergoing any process is

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc., energies}}}$$

Then the energy balance for the ball for a process from point 1 to point 2 becomes

$$-w_{friction} = (\text{ke}_2 + \text{pe}_2) - (\text{ke}_1 + \text{pe}_1)$$

or

$$\frac{V_1^2}{2} + gz_1 = \frac{V_2^2}{2} + gz_2 + w_{friction}$$

since there is no energy transfer by heat or mass, and no change in the internal energy of the ball (the heat generated by frictional heating is dissipated to the surrounding air). The frictional work term $w_{friction}$ is often expressed as e_{loss} to represent the loss (conversion) of mechanical energy into thermal energy.

For the idealized case of frictionless motion, the last relation reduces to

$$\frac{V_1^2}{2} + gz_1 = \frac{V_2^2}{2} + gz_2 \qquad \text{or} \qquad \frac{V^2}{2} + gz = C = \text{constant}$$

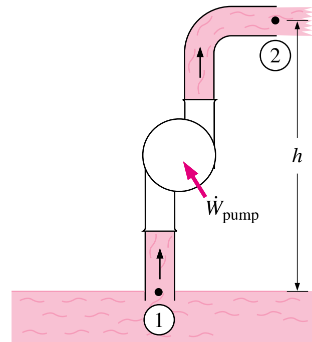
where the value of the constant is $C = gh$. That is, *when the frictional effects are negligible, the sum of the kinetic and potential energies of the ball remains constant.*

Discussion This is certainly a more intuitive and convenient form of the conservation of energy equation for this and other similar processes such as the swinging motion of the pendulum of a wall clock. The relation obtained is analogous to the Bernoulli equation derived in the next section.

Most processes encountered in practice involve only certain forms of energy, and in such cases it is more convenient to work with the simplified versions of the energy balance. For systems that involve only *mechanical forms of energy* and its transfer as *shaft work,* the conservation of energy principle can be expressed conveniently as

$$E_{\text{mech, in}} - E_{\text{mech, out}} = \Delta E_{\text{mech, system}} + E_{\text{mech, loss}} \qquad \textbf{(12–9)}$$

where $E_{\text{mech, loss}}$ represents the conversion of mechanical energy to thermal energy due to irreversibilities such as friction. For a system in steady operation, the mechanical energy balance becomes $\dot{E}_{\text{mech, in}} = \dot{E}_{\text{mech, out}} + \dot{E}_{\text{mech, loss}}$ (Fig. 12–7).



Steady flow
$\mathcal{V}_1 = \mathcal{V}_2$
$z_2 = z_1 + h$
$P_1 \cong P_2 \cong P_{\text{atm}}$

$$\dot{E}_{\text{mech, in}} = \dot{E}_{\text{mech, out}} + \dot{E}_{\text{mech, loss}}$$
$$\dot{W}_{\text{pump}} + \dot{m}gz_1 = \dot{m}gz_2 + \dot{E}_{\text{mech, loss}}$$
$$\dot{W}_{\text{pump}} = \dot{m}gh + \dot{E}_{\text{mech, loss}}$$

FIGURE 12–7
Most fluid flow problems involve mechanical forms of energy only, and such problems are conveniently solved by using a *mechanical energy* balance.

12–2 ▪ THE BERNOULLI EQUATION

The **Bernoulli equation** is *an approximate relation between pressure, velocity, and elevation*, and is valid in *regions of steady, incompressible flow where net frictional forces are negligible* (Fig. 12–8). Despite its simplicity, it has proven to be a very powerful tool in fluid mechanics. In this section, we derive the Bernoulli equation by applying the *conservation of linear momentum principle*, and we demonstrate both its usefulness and its limitations.

The key approximation in the derivation of the Bernoulli equation is that *viscous effects are negligibly small compared to inertial, gravitational, and pressure effects*. Since all fluids have viscosity (there is no such thing as an "inviscid fluid"), this approximation cannot be valid for an entire flow field of practical interest. In other words, we cannot apply the Bernoulli equation *everywhere* in a flow, no matter how small the fluid's viscosity. However, it turns out that the approximation *is* reasonable in certain *regions* of many practical flows. We refer to such regions as *inviscid regions of flow*, and we stress that they are *not* regions where the fluid itself is inviscid or frictionless, but rather they are regions where net viscous or frictional forces are negligibly small compared to other forces acting on fluid particles.

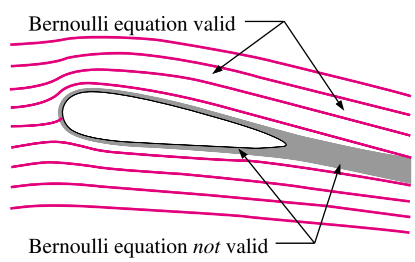


FIGURE 12–8
The *Bernoulli equation* is an *approximate* equation that is valid only in *inviscid regions of flow* where net viscous forces are negligibly small compared to inertial, gravitational, or pressure forces. Such regions occur outside of *boundary layers* and *wakes.*

Care must be exercised when applying the Bernoulli equation since it is an approximation that applies only to inviscid regions of flow. In general, frictional effects are always important very close to solid walls (*boundary layers*) and directly downstream of bodies (*wakes*). Thus, the Bernoulli approximation is typically useful in flow regions outside of boundary layers and wakes, where the fluid motion is governed by the combined effects of pressure and gravity forces.

The motion of a particle and the path it follows are described by the *velocity vector* as a function of time and space coordinates and the initial position of the particle. When the flow is *steady* (no change with time at a specified location), all particles that pass through the same point follow the same path (which is the *streamline*), and the velocity vectors remain tangent to the path at every point.

Acceleration of a Fluid Particle

Often it is convenient to describe the motion of a particle in terms of its distance s from the origin together with the radius of curvature along the streamline. The velocity of the particle is related to the distance by $\mathcal{V} = ds/dt$, which may vary along the streamline. In two-dimensional flow, the acceleration can be decomposed into two components: *streamwise acceleration* a_s along the streamline and *normal acceleration* a_n in the direction normal to the streamline, which is given as $a_n = \mathcal{V}^2/R$. Note that streamwise acceleration is due to a change in speed along a streamline, and normal acceleration is due to a change in direction. For particles that move along a *straight path,* $a_n = 0$ since the radius of curvature is infinity and thus there is no change in direction. The Bernoulli equation results from a force balance along a streamline.

One may be tempted to think that acceleration is zero in steady flow since acceleration is the rate of change of velocity with time, and in steady flow there is no change with time. Well, a garden hose nozzle tells us that this understanding is not correct. Even in steady flow and thus constant mass flow rate, water accelerates through the nozzle (Fig. 12–9). *Steady* simply means *no change with time at a specified location,* but the value of a quantity may change from one location to another. In the case of a nozzle, the velocity of water remains constant at a specified point, but it changes from the inlet to the exit (water accelerates along the nozzle).

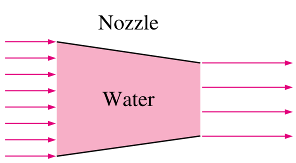


FIGURE 12–9
During steady flow, a fluid may not accelerate in time at a fixed point, but it may accelerate in space.

Mathematically, this can be expressed as follows: We take the velocity $\mathcal{V}$ to be a function of s and t. Taking the total differential of $\mathcal{V}(s, t)$ and dividing both sides by dt give

$$d\mathcal{V} = \frac{\partial \mathcal{V}}{\partial s}\, ds + \frac{\partial \mathcal{V}}{\partial t}\, dt \qquad \text{and} \qquad \frac{d\mathcal{V}}{dt} = \frac{\partial \mathcal{V}}{\partial s}\frac{ds}{dt} + \frac{\partial \mathcal{V}}{\partial t} \tag{12–10a, b}$$

In steady flow $\partial \mathcal{V}/\partial t = 0$ and thus $\mathcal{V} = \mathcal{V}(s)$, and the acceleration in the s direction becomes

$$a_s = \frac{d\mathcal{V}}{dt} = \frac{\partial \mathcal{V}}{\partial s}\frac{ds}{dt} = \frac{\partial \mathcal{V}}{\partial s}\mathcal{V} = \mathcal{V}\frac{d\mathcal{V}}{ds} \tag{12–11}$$

Therefore, acceleration in steady flow is due to the change of velocity with position.

Derivation of the Bernoulli Equation

Consider the motion of a fluid particle in a flow field in steady flow. Applying Newton's second law (which is referred to as the *conservation of linear momentum* relation in fluid mechanics) in the s direction on a particle moving along a streamline gives

$$\sum F_s = ma_s \tag{12–12}$$

In regions of flow where net fictional forces are negligible, the significant forces acting in the s direction are the pressure (acting on both sides) and the component of the weight of the particle in the s direction (Fig. 12–10). Therefore, Eq. 12–12 becomes

$$P\, dA - (P + dP)\, dA - W \sin\theta = m\mathcal{V}\frac{d\mathcal{V}}{ds} \tag{12–13}$$

where θ is the angle between the normal of the streamline and the vertical z axis at that point, $m = \rho V = \rho\, dA\, ds$ is the mass, $W = mg = \rho g\, dA\, ds$ is the weight of the fluid particle, and $\sin\theta = dz/ds$. Substituting,

$$-dP\, dA - \rho g\, dA\, ds\frac{dz}{ds} = \rho\, dA\, ds\, \mathcal{V}\frac{d\mathcal{V}}{ds} \tag{12–14}$$

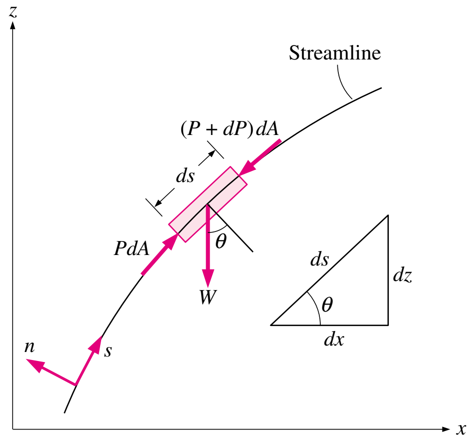


FIGURE 12–10
The forces acting on a fluid particle along a streamline.

Canceling dA from each term and simplifying,

$$-dP - \rho g\, dz = \rho \mathcal{V}\, d\mathcal{V} \qquad \textbf{(12–15)}$$

Noting that $\mathcal{V}\, d\mathcal{V} = \frac{1}{2} d(\mathcal{V}^2)$ and dividing each term by ρ gives

$$\frac{dP}{\rho} + \frac{1}{2} d(\mathcal{V}^2) + g\, dz = 0 \qquad \textbf{(12–16)}$$

Integrating (Fig. 12–11),

Steady flow: $$\int \frac{dP}{\rho} + \frac{\mathcal{V}^2}{2} + gz = \text{constant} \quad \text{(along a streamline)} \qquad \textbf{(12–17)}$$

since the last two terms are exact differentials. In the case of incompressible flow, the first term also becomes an exact differential, and its integration gives

Steady, incompressible flow: $$\frac{P}{\rho} + \frac{\mathcal{V}^2}{2} + gz = \text{constant} \qquad \text{(kJ/kg)} \qquad \textbf{(12–18)}$$

(Steady flow)

General:

$$\int \frac{dP}{\rho} + \frac{\mathcal{V}^2}{2} + gz = \text{constant}$$

Incompressible flow (ρ = constant):

$$\frac{P}{\rho} + \frac{\mathcal{V}^2}{2} + gz = \text{constant}$$

FIGURE 12–11
The Bernoulli equation is derived assuming incompressible flow, and thus it should not be used for flows with significant compressibility effects.

This is the famous **Bernoulli equation,** which is commonly used in fluid mechanics for steady, incompressible flow in inviscid regions of flow. The value of the constant can be evaluated at any point on the streamline where the pressure, density, velocity, and elevation are known. The Bernoulli equation can also be written between any two points on the same streamline as

Steady, incompressible flow: $$\frac{P_1}{\rho} + \frac{\mathcal{V}_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{\mathcal{V}_2^2}{2} + gz_2 \qquad \textbf{(12–19)}$$

The Bernoulli equation is obtained from the conservation of momentum for a fluid particle moving along a streamline. It can also be obtained from the *first law of thermodynamics* applied to a steady-flow system, as shown later in this chapter.

The Bernoulli equation was first stated verbally in a textbook by Daniel Bernoulli in 1738 and was derived later by Leonhard Euler in 1755. We recognize $\mathcal{V}^2/2$ as *kinetic energy,* gz as *potential energy,* and P/ρ as *flow energy* per unit mass. Therefore, the Bernoulli equation can be viewed as an expression of *mechanical energy balance* and can be stated as follows (Fig. 12–12):

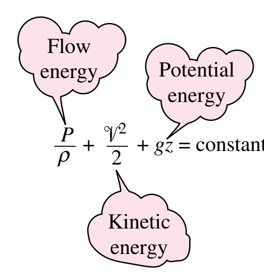


FIGURE 12–12
The Bernoulli equation states that the sum of the kinetic, potential, and flow energies of a fluid particle is constant along a streamline during steady flow.

The sum of the kinetic, potential, and flow energies of a fluid particle is constant along a streamline during steady flow when the compressibility and frictional effects are negligible.

The kinetic, potential, and flow energies are the mechanical forms of energy, as discussed earlier, and the Bernoulli equation can be viewed as the "conservation of mechanical energy principle." This is equivalent to the general conservation of energy principle for systems that do not involve any conversion of mechanical energy and thermal energy to each other, and thus the mechanical energy and thermal energy are conserved separately. The Bernoulli equation states that during steady, incompressible flow with negligible friction, the various forms of mechanical energy are converted to each other, but their sum remains constant. In other words, there is no dissipation of mechanical energy during such flows since there is no friction that converts mechanical energy to sensible thermal (internal) energy.

Recall that energy is transferred to a system as work when a force is applied to a system through a distance. In the light of Newton's second law of motion, the Bernoulli equation can also be viewed as *the work done by the pressure and gravity forces on the fluid particle is equal to the increase in the kinetic energy of the particle.*

Despite the highly restrictive approximations used in its derivation, the Bernoulli equation is commonly used in practice since a variety of practical fluid flow problems can be analyzed to reasonable accuracy with it. This is because many flows of practical engineering interest are steady (or at least steady in the mean), compressibility effects are relatively small, and net frictional forces are negligible in regions of interest in the flow.

Unsteady Compressible Flow

Similarly, using both terms in the acceleration expression (Eq. 12–10), it can be shown that the Bernoulli equation for *unsteady, compressible flow* is

Unsteady, compressible flow:
$$\int \frac{dP}{\rho} + \int \frac{\partial \mathcal{V}}{\partial t}\, ds + \frac{\mathcal{V}^2}{2} + gz = \text{constant} \qquad \textbf{(12–20)}$$

Force Balance across Streamlines

It is left as an exercise to show that a force balance in the direction n normal to the streamline yields the following relation applicable *across* the streamlines for steady incompressible flow:

$$\frac{P}{\rho} + \int \frac{\mathcal{V}^2}{R}\, dn + gz = \text{constant} \qquad \text{(across streamlines)} \qquad \textbf{(12–21)}$$

For flow along a straight line, $R \rightarrow \infty$ and thus the relation above reduces to $P/\rho + gz =$ constant or $P = -\rho gz +$ constant, which is an expression for the variation of hydrostatic pressure with vertical distance for a stationary fluid body. Therefore, the variation of pressure with elevation in steady incompressible flow along a straight line is the same as that in the stationary fluid (Fig. 12–13).

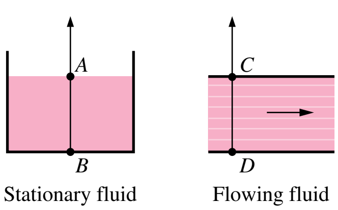


FIGURE 12–13
The variation of pressure with elevation in steady incompressible flow along a straight line is the same as that in the stationary fluid (but this is not the case for a curved flow section).

Static, Dynamic, and Stagnation Pressures

The Bernoulli equation states that the sum of the flow, kinetic, and potential energies of a fluid particle along a streamline is constant. Therefore, the kinetic and potential energies of the fluid can be converted to flow energy (and vice versa) during flow, causing the pressure to change. This phenomenon can be made more visible by multiplying the Bernoulli equation by the density ρ,

$$P + \rho \frac{\mathcal{V}^2}{2} + \rho g z = \text{constant} \qquad \text{(kPa)} \tag{12–22}$$

Each term in this equation has pressure units, and thus each term represents some kind of pressure:

- P is the **static pressure** (it does not incorporate any dynamic effects); it represents the actual pressure of the fluid. This is the same as the pressure used in thermodynamics and property tables.
- $\rho\mathcal{V}^2/2$ is the **dynamic pressure;** it represents the pressure rise when the fluid in motion is brought to a stop isentropically.
- $\rho g z$ is the **hydrostatic pressure,** which is not pressure in a real sense since its value depends on the reference level selected; it accounts for the elevation effects, i.e., of fluid weight on pressure.

The sum of the static, dynamic, and hydrostatic pressures is called the **total pressure.** Therefore, the Bernoulli equation states that *the total pressure along a streamline is constant.*

The sum of the static and dynamic pressures is called the **stagnation pressure,** and it is expressed as

$$P_{\text{stag}} = P + \rho \frac{\mathcal{V}^2}{2} \qquad \text{(kPa)} \tag{12–23}$$

The stagnation pressure represents the pressure at a point where the fluid is brought to a complete stop isentropically. The static, dynamic, and stagnation pressures are shown in Fig. 12–14. When static and stagnation pressures are measured at a specified location, the fluid velocity at that location can be calculated from

$$\mathcal{V} = \sqrt{\frac{2(P_{\text{stag}} - P)}{\rho}} \tag{12–24}$$

Equation 12–24 is useful in the measurement of flow velocity when a combination of a static pressure tap and a Pitot tube is used, as illustrated in Fig. 12–14. A **static pressure tap** is simply a small hole drilled into a wall such that the plane of the hole is parallel to the flow direction. It measures the static pressure. A **Pitot tube** is a small tube with its open end aligned *into* the flow so as to sense the full impact pressure of the flowing fluid. It measures the stagnation pressure. In situations in which the static and stagnation pressure of a flowing *liquid* are greater than atmospheric pressure, a vertical transparent tube called a **piezometer tube** (or simply a **piezometer**) can be attached to the pressure tap and to the Pitot tube, as sketched. The liquid rises in the piezometer tube to a column height (*head*) that is proportional to the pressure being measured. If the pressures to be measured are below atmospheric, or if measuring pressures in *gases*, piezometer tubes do not work. However, the static pressure tap and Pitot tube can still be used, but they must be connected to some other kind of pressure measurement device such as a U-tube manometer or a pressure transducer.

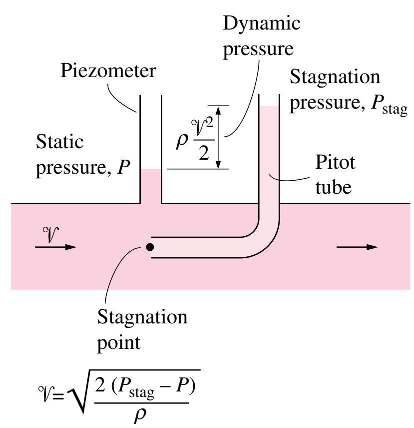


FIGURE 12–14
The static, dynamic, and stagnation pressures.

When the static pressure is measured by drilling a hole in the tube wall, care must be exercised to ensure that the opening of the hole is flush with the wall surface, with no extrusions before or after the hole (Fig. 12–15). Otherwise the reading will incorporate some dynamic effects, and thus it will be in error.

When a stationary body is immersed in a flowing stream, the fluid is brought to a stop at the nose of the body (the **stagnation point**). The flow streamline that extends from far upstream to the stagnation point is called the **stagnation streamline** (Fig. 12–16). For a two-dimensional flow in the x-y plane, the stagnation point is actually a *line* parallel the z-axis, and the stagnation streamline is actually a *surface* that separates fluid that flows *over* the body from fluid that flows *under* the body. In an incompressible flow, the fluid decelerates nearly isentropically from its freestream value to zero at the stagnation point, and the pressure at the stagnation point is thus the stagnation pressure.

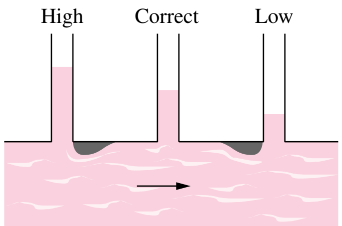


FIGURE 12–15
Careless drilling of the static pressure tap may result in erroneous reading of the static pressure.

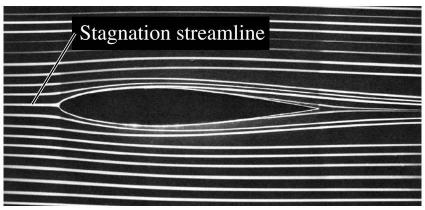


FIGURE 12–16
Streaklines produced by colored fluid introduced upstream of an airfoil; since the flow is steady, the streaklines are the same as streamlines and pathlines. The stagnation streamline is marked. (Courtesy, ONERA).

Limitations on the Use of the Bernoulli Equation

The Bernoulli equation is one of the most frequently used and misused equations in fluid mechanics. Its versatility, simplicity, and ease of use make it a very valuable tool for use in analysis, but the same attributes also make it very tempting to misuse. Therefore, it is important to understand the restrictions on its applicability and observe the limitations on its use, as explained below:

1. **Steady flow** The first limitation on the Bernoulli equation is that it is applicable to *steady flow.* Therefore, it should not be used during the transient start-up and shut-down periods, or during periods of change in the flow conditions. Note that there is an unsteady form of the Bernoulli equation, discussion of which is beyond the scope of the present text (see Panton, 1996).

2. **Frictionless flow** Every flow involves some friction, no matter how small, and *frictional effects* may or may not be negligible. The situation is complicated even more by the amount of error that can be tolerated. In general, frictional effects are negligible for short flow sections with large cross sections, especially at low flow velocities. Frictional effects are usually significant in long and narrow flow passages, in the wake region downstream of an object, and in *diverging flow sections* such as diffusers because of the increased possibility of the fluid separating from the walls in such geometries. Frictional effects are also significant near solid surfaces, and thus the Bernoulli equation is usually applicable along a streamline in the core region of the flow, but not along a streamline close to the surface (Fig. 12–17).

   A component that disturbs the streamlined structure of flow and thus causes considerable mixing and back flow such as a sharp entrance of a tube or a partially closed valve in a flow section can make the Bernoulli equation inapplicable.

3. **No shaft work** The Bernoulli equation was derived from a force balance on a particle moving along a streamline. Therefore, the Bernoulli equation is not applicable in a flow section that involves a pump, turbine, fan, or any other machine or impeller since such devices destroy the streamlines and carry out energy interactions with the fluid

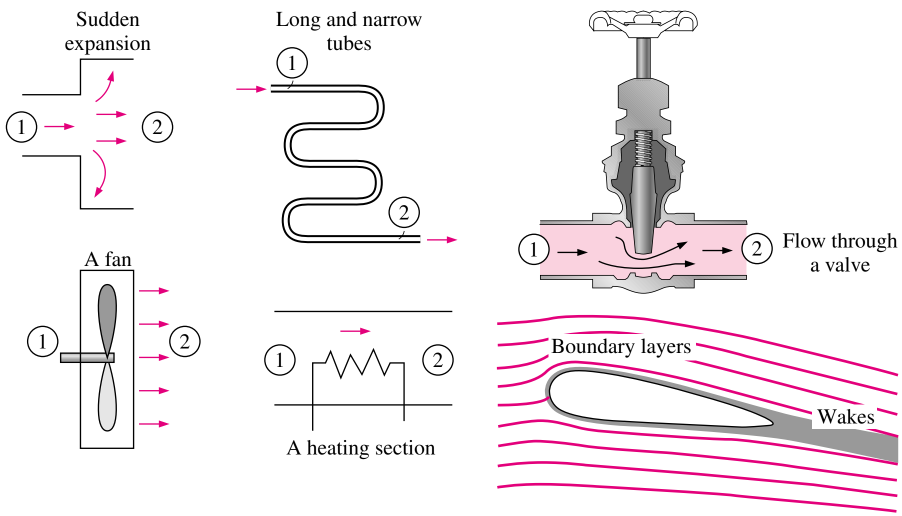


FIGURE 12–17
Frictional effects and components that disturb the streamlined structure of flow in a flow section make the Bernoulli equation invalid.

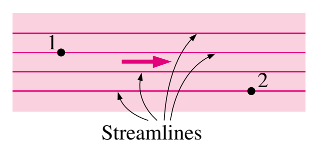
particles. When the flow section considered involves any of these devices, the energy equation should be used instead to account for the shaft work input or output. However, the Bernoulli equation can still be applied to a flow section prior to or past a machine (assuming, of course, that the other restrictions on its use are satisfied). In such cases, the Bernoulli constant changes from upstream to downstream of the device.

4. **Incompressible flow** One of the assumptions used in the derivation of the Bernoulli equation is that ρ = constant and thus the flow is incompressible. This condition is satisfied by liquids and also by gases at Mach numbers less than about 0.3 since compressibility effects and thus density variations of gases are negligible at such relatively low velocities. Note that there is a compressible form of the Bernoulli equation (Eq. 12–20).

5. **No heat transfer** The density of a gas is inversely proportional to temperature, and thus the Bernoulli equation should not be used for flow sections that involve significant temperature change such as heating or cooling sections.

6. **Flow along a streamline** Strictly speaking, the Bernoulli equation $P/\rho + \mathcal{V}^2/2 + gz = C$ is applicable along a streamline, and the value of the constant C, in general, is different for different streamlines. But when a region of the flow is *irrotational,* and thus there is no *vorticity* in the flow field, the value of the constant C remains the same for all streamlines, and, therefore, the Bernoulli equation becomes applicable *across* streamlines as well (Fig. 12–18). Therefore, we do not need to be concerned about the streamlines when the flow is irrotational, and we can apply the Bernoulli equation between any two points in the irrotational region of the flow.

We derived the Bernoulli equation by considering two-dimensional flow in the x-z plane for simplicity, but the equation is valid for general three-dimensional flow as well, as long as it is applied along the same streamline.



$$\frac{P_1}{\rho} + \frac{\mathcal{V}_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{\mathcal{V}_2^2}{2} + gz_2$$

FIGURE 12–18
When the flow is irrotational, the Bernoulli equation becomes applicable between any two points along the flow (not just on the same streamline).

We should always keep in mind the assumptions used in the derivation of the Bernoulli equation and make sure that they are not violated.

Hydraulic Grade Line (HGL) and Energy Grade Line (EGL)

It is often convenient to represent the level of mechanical energy graphically using *heights* to facilitate visualization of the various terms of the Bernoulli equation. This is done by dividing each term of the Bernoulli equation by g to give

$$\frac{P}{\rho g} \square \frac{\mathcal{V}^2}{2g} \square z \square H \square \text{constant} \qquad \text{(m)} \tag{12–25}$$

Each term in this equation has the dimension of length and represents some kind of "head" of a flowing fluid as follows:

- $P/\rho g$ is the **pressure head;** it represents the height of a fluid column that produces the static pressure P.
- $\mathcal{V}^2/2g$ is the **velocity head;** it represents the elevation needed for a fluid to reach the velocity $\mathcal{V}$ during frictionless free fall.
- z is the **elevation head;** it represents the potential energy of the fluid.

Also, H is the **total head** for the flow. Therefore, the Bernoulli equation can be expressed in terms of heads as: *the sum of the pressure, velocity, and elevation heads along a streamline is constant during steady flow when the compressibility and frictional effects are negligible* (Fig. 12–19).

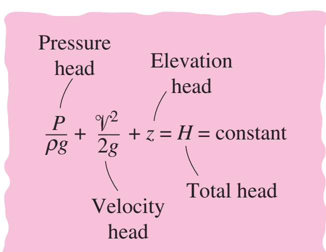


FIGURE 12–19
An alternative form of the Bernoulli equation is expressed in terms of heads as *the sum of the pressure, velocity, and elevation heads is constant along a streamline.*

If a piezometer (measures static pressure) is tapped into a pipe, as shown in Fig. 12–20, the liquid would rise to a height of $P/\rho g$ above the pipe center. The *hydraulic grade line* (HGL) is obtained by doing this at several locations along the pipe and drawing a line through the liquid levels in piezometers. The vertical distance above the pipe center is a measure of pressure within the pipe. Similarly, if a pitot tube (measures static □ dynamic pressure) is tapped into a pipe, the liquid would rise to a height of $P/\rho g \square \mathcal{V}^2/2g$ above the pipe center, or a distance of $\mathcal{V}^2/2g$ above the HGL. The *energy grade line* (EGL) is obtained by doing this at several locations along the pipe and drawing a line through the liquid levels in pitot tubes.

Noting that the fluid also has elevation head z (unless the reference level is taken to be the centerline of the pipe), the HGL and EGL can be defined as follows: The line that represents the sum of the static pressure and the elevation heads $P/\rho g \square z,$ is called the **hydraulic grade line.** The line that represents the total head of the fluid, $P/\rho g \square \mathcal{V}^2/2g \square z,$ is called the **energy grade line.** The difference between the heights of EGL and HGL is equal to the dynamic head, $\mathcal{V}^2/2g$. We note the following about the HGL and EGL:

- For *stationary bodies* such as reservoirs or lakes, the EGL and HGL coincide with the free surface of the liquid. The elevation of the free surface z in such cases represents both the EGL and the HGL since the velocity is zero and the static pressure (gage) is zero.

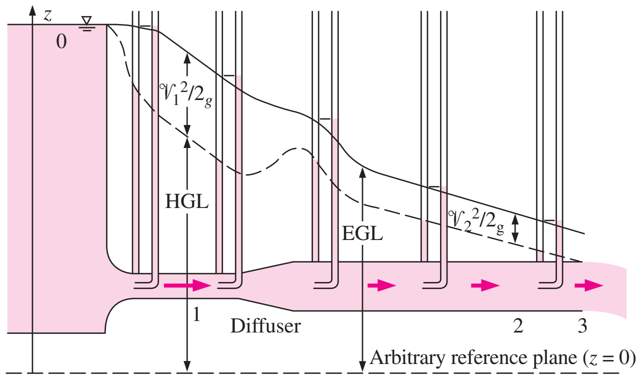


FIGURE 12–20

The *hydraulic grade line* (HGL) and the *energy grade line* (EGL) for free discharge from a reservoir through a horizontal pipe with a diffuser. At point 0 (at the liquid surface), EGL and HGL are even with the liquid surface since there is no flow there. HGL decreases rapidly as the liquid accelerates into the pipe; however EGL decreases very slowly through the well-rounded pipe inlet. EGL declines continually along the flow direction due to friction and other irreversible losses in the flow. EGL cannot increase in the flow direction unless energy is supplied to the fluid. HGL can rise or fall in the flow direction, but can never exceed EGL. HGL rises in the diffuser section as the velocity decreases, and the static pressure recovers somewhat; the total pressure does *not* recover, however, and EGL decreases through the diffuser. The difference between EGL and HGL is $\mathcal{V}_1^2/2g$ at point 1, and $\mathcal{V}_2^2/2g$ at point 2. Since $\mathcal{V}_1$ □ $\mathcal{V}_2$, the difference between the two grade lines is larger at point 1 than at point 2. The downward slope of both grade lines is larger for the smaller diameter section of pipe since the frictional head loss is greater. Finally, HGL decays to the liquid surface at the outlet since the pressure there is atmospheric. However, EGL is still higher than HGL by the amount $\mathcal{V}_2^2/2g$ since $\mathcal{V}_3$ □ $\mathcal{V}_2$ at the outlet.

- The EGL is always a distance $\mathcal{V}^2/2g$ above the HGL. These two lines approach each other as the velocity decreases, and they diverge as the velocity increases. The height of the HGL decreases as the velocity increases, and vice versa.
- In an *idealized Bernoulli-type flow,* EGL is horizontal and its height remains constant. This would also be the case for HGL when the flow velocity is constant (Fig. 12–21).
- For *open channel flow,* the HGL coincides with the free surface of the liquid, and the EGL is a distance $\mathcal{V}^2/2g$ above the free surface.
- At a *pipe exit,* the pressure head is zero (atmospheric pressure) and thus the HGL coincides with the pipe exit.
- The *mechanical energy loss* due to frictional effects (conversion to thermal energy) causes the EGL and HGL to slope downward in the direction of flow. The slope is a measure of the pipe loss (discussed in detail in Chap. 14). A component that generates significant frictional

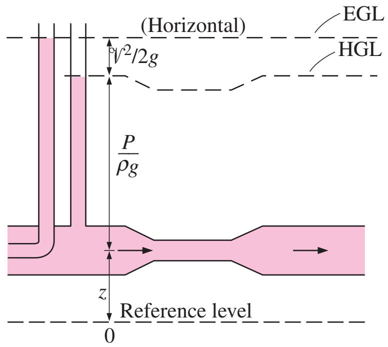


FIGURE 12–21

In an idealized Bernoulli-type flow, EGL is horizontal and its height remains constant. But this is not the case for HGL when the flow velocity varies along the flow.

effects such as a valve causes a sudden drop in both *EGL* and *HGL* at that location.

- A *steep jump* occurs in EGL and HGL whenever mechanical energy is added to the fluid (by a pump, for example). Likewise, a *steep drop* occurs in EGL and HGL whenever mechanical energy is removed from the fluid (by a turbine, for example), as shown in Fig. 12–22.
- The pressure (gage) of a fluid is zero at locations where the HGL *intersects* the fluid. The pressure in a flow section that lies above the HGL is negative, and the pressure in a section that lies below the HGL is positive (Fig. 12–23). Therefore, an accurate drawing of a piping system and the HGL can be used to determine the regions where the pressure in the pipe is negative (below the atmospheric pressure).

The last remark enables us to avoid situations in which the pressure drops below the vapor pressure of the liquid (which causes *cavitation,* as discussed in Chap. 10). Proper consideration is necessary in the placement of a liquid pump to ensure that the suction side pressure does not fall too low, especially at elevated temperatures where vapor pressure is higher than it is at low temperatures.

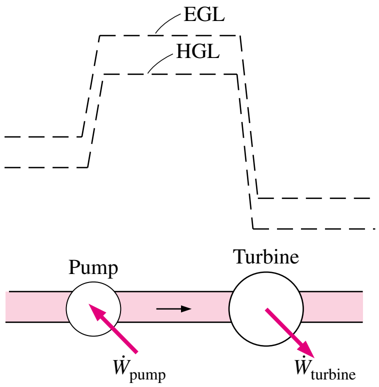


FIGURE 12–22
A *steep jump* occurs in EGL and HGL whenever mechanical energy is added to the fluid by a pump, and a *steep drop* occurs whenever mechanical energy is removed from the fluid by a turbine.

12–3 ▪ APPLICATIONS OF THE BERNOULLI EQUATION

In Section 12–2, we discussed the fundamental aspects of the Bernoulli equation. In this section, we will demonstrate its use in a wide range of applications through examples.

EXAMPLE 12–3 **Spraying Water into the Air**

Water is flowing from a hose attached to a water main at 400 kPa gage (Fig. 12–24). A child places his thumb to cover most of the hose outlet, increasing the pressure upstream of his thumb, causing a thin jet of high-speed water to emerge. If the hose is held upward, what is the maximum height that the jet could achieve?

SOLUTION Water from a hose attached to the water main is sprayed into the air. The maximum height the water jet can rise is to be determined.
Assumptions **1** The flow exiting into the air is steady, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The water pressure in

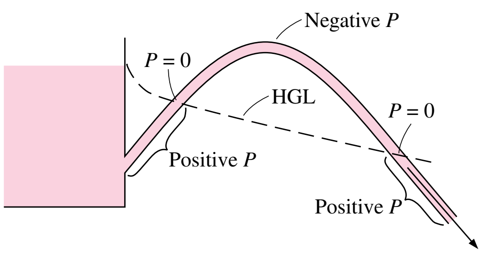


FIGURE 12–23
The pressure (gage) of a fluid is zero at locations where the HGL *intersects* the fluid, and the pressure is negative (vacuum) in a flow section that lies above the HGL.

the hose near the outlet is equal to the water main pressure. **3** The surface tension effects are negligible. **4** The friction between the water and air is negligible. **5** The irreversibilities that may occur at the outlet of the hose due to abrupt expansion are negligible.

Properties We take the density of water to be 1000 kg/m^3.

Analysis This problem involves the conversion of flow, kinetic, and potential energies to each other without involving any pumps, turbines, and wasteful components with large frictional losses, and thus it is suitable for the use of the Bernoulli equation. The water height will be maximum under the stated assumptions. The velocity inside the hose is relatively low ($\mathcal{V}_1 \cong 0$) and we take the hose outlet as the reference level ($z_1 = 0$). At the top of the water trajectory $\mathcal{V}_2 = 0$, and atmospheric pressure pertains. Then the Bernoulli equation simplifies to

$$\frac{P_1}{\rho g}+\overset{0}{\frac{\mathcal{V}_1^2}{2g}}+\overset{0}{z_1}=\frac{P_2}{\rho g}+\overset{0}{\frac{\mathcal{V}_2^2}{2g}}+z_2 \quad\to\quad \frac{P_1}{\rho g}=\frac{P_{atm}}{\rho g}+z_2$$

Solving for z_2 and substituting,

$$z_2=\frac{P_1-P_{atm}}{\rho g}=\frac{P_{1,\,gage}}{\rho g}=\frac{400\text{ kPa}}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)}\left(\frac{1000\text{ N/m}^2}{1\text{ kPa}}\right)\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)$$

$$= \mathbf{40.8\text{ m}}$$

Therefore, the water jet can rise as high as 40.8 m into the sky in this case.

Discussion The result obtained by the Bernoulli equation represents the upper limit and should be interpreted accordingly. It tells us that the water cannot possibly rise more than 40.8 m, and, in all likelihood, the rise will be much less than 40.8 m due to irreversible losses that we neglected.

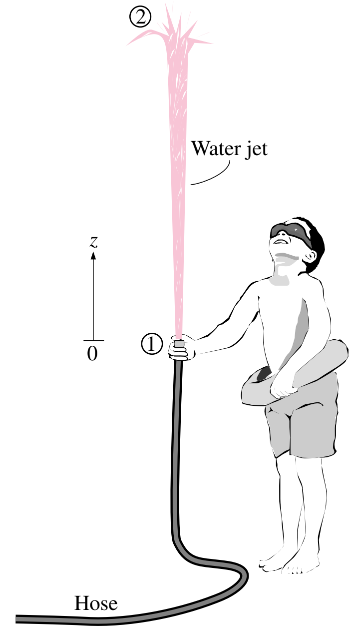


FIGURE 12–24
Schematic for Example 12–3.

EXAMPLE 12–4 Water Discharge from a Large Tank

A large tank open to the atmosphere is filled with water to a height of 5 m from the bottom (Fig. 12–25). A tap near the bottom of the tank is now opened, and water flows out from the smooth and rounded outlet. Determine the water velocity at the outlet.

SOLUTION A tap near the bottom of a tank is opened. The exit velocity of water from the tank is to be determined.

Assumptions **1** The flow is incompressible and irrotational (except very close to the walls). **2** The water drains slowly enough that the flow can be approximated as steady (actually quasi-steady when the tank begins to drain).

Analysis This problem involves the conversion of flow, kinetic, and potential energies to each other without involving any pumps, turbines, and wasteful components with large frictional losses, and thus it is suitable for the use of the Bernoulli equation. We take point 1 to be at the free surface of water so that $P_1 = P_{atm}$ (open to the atmosphere), $\mathcal{V}_1 \cong 0$ (the tank is large relative to the outlet), and $z_1 = 5$ m and $z_2 = 0$ (we take the reference level at the center of the outlet). Also, $P_2 = P_{atm}$ (water discharges into the atmosphere). Then the Bernoulli equation simplifies to

$$\frac{P_1}{\rho g}+\overset{0}{\frac{\mathcal{V}_1^2}{2g}}+z_1=\frac{P_2}{\rho g}+\frac{\mathcal{V}_2^2}{2g}+\overset{0}{z_2} \quad\to\quad z_1=\frac{\mathcal{V}_2^2}{2g}$$

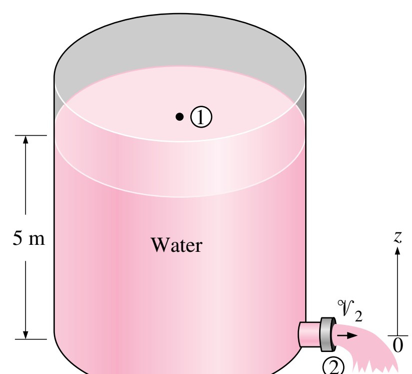


FIGURE 12–25
Schematic for Example 12–4.

Solving for $\mathcal{V}_2$ and substituting,

$$\mathcal{V}_2 = \sqrt{2gz_1} = \sqrt{2(9.81\ \text{m/s}^2)(5\ \text{m})} = \mathbf{9.9\ m/s}$$

The relation $\mathcal{V} = \sqrt{2gz}$ is called the **Toricelli equation.**

Therefore, the water leaves the tank with an initial velocity of 9.9 m/s. This is the same velocity that would manifest if a solid were dropped a distance of 5 m in the absence of air friction drag. (What would the velocity be if the tap were at the bottom of the tank instead of on the side?)

Discussion If the orifice were sharp-edged instead of rounded, then the flow would be disturbed, and the velocity would be less than 9.9 m/s, especially near the edges. Care must be exercised when attempting to apply the Bernoulli equation to situations where abrupt expansions or contractions occur since the friction and flow disturbance in such cases may not be negligible.

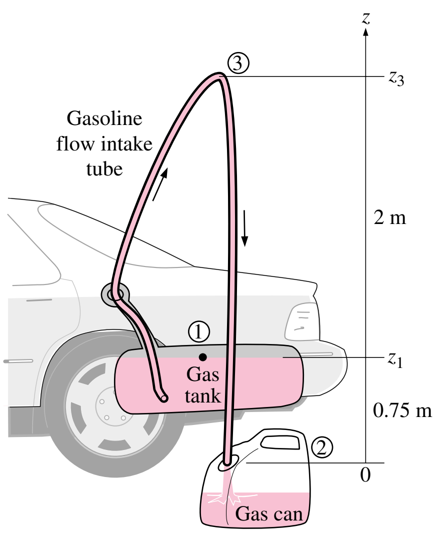


FIGURE 12–26
Schematic for Example 12–5.

EXAMPLE 12–5 Siphoning out Gasoline from a Fuel Tank

During a trip to the beach (P_{atm} = 1 atm = 101.3 kPa), a car runs out of gasoline, and it becomes necessary to siphon gas out of the car of a good Samaritan (Fig. 12–26). The siphon is a small-diameter hose, and to start the siphon it is necessary to insert one siphon end in the full gas tank, fill the hose with gasoline via suction, and then place the other end in a gas can below the level of the gas tank. The difference in pressure between point 1 (at the free surface of the gasoline in the tank) and point 2 (at the outlet of the tube) will cause the liquid to flow from the higher to the lower elevation. Point 2 is located 0.75 m below point 1 in this case, and point 3 is located 2 m above point 1. The siphon diameter is 4 mm, and frictional losses in the siphon are to be disregarded. Determine (*a*) the minimum time to withdraw 4 L of gasoline from the tank to the can and (*b*) the pressure at point 3. The density of gasoline is 750 kg/m³.

SOLUTION Gasoline is to be siphoned from a tank. The time it takes to withdraw 4 L of gasoline and the pressure at the highest point in the system are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** Even though the Bernoulli equation is not valid through the pipe because of frictional losses, we employ the Bernoulli equation anyway in order to obtain a *best-case estimate.* **3** The change in the gasoline surface level inside the tank is negligible compared to elevations z_1 and z_2 during the siphoning period.

Properties The density of gasoline is given to be 750 kg/m³.

Analysis (*a*) We take point 1 to be at the free surface of gasoline in the tank so that $P_1 = P_{atm}$ (open to the atmosphere), $\mathcal{V}_1 \cong 0$ (the tank is large relative to the tube diameter), and $z_2 = 0$ (point 2 is taken as the reference level). Also, $P_2 = P_{atm}$ (gasoline discharges into the atmosphere). Then the Bernoulli equation simplifies to

$$\frac{P_1}{\rho g} + \frac{\mathcal{V}_1^2}{2g}\overset{0}{} + z_1 = \frac{P_2}{\rho g} + \frac{\mathcal{V}_2^2}{2g} + z_2\overset{0}{} \quad \rightarrow \quad z_1 = \frac{\mathcal{V}_2^2}{2g}$$

Solving for $\mathcal{V}_2$ and substituting,

$$\mathcal{V}_2 = \sqrt{2gz_1} = \sqrt{2(9.81\ \text{m/s}^2)(0.75\ \text{m})} = 3.84\ \text{m/s}$$

The cross-sectional area of the tube and the flow rate of gasoline are

$$A = \pi D^2/4 = \pi(5 \times 10^{-3}\ \text{m})^2/4 = 1.96 \times 10^{-5}\ \text{m}^2$$
$$\dot{V} = \mathcal{V}_2 A = (3.84\ \text{m/s})(1.96 \times 10^{-5}\ \text{m}^2) = 7.53 \times 10^{-5}\ \text{m}^3/\text{s} = 0.0753\ \text{L/s}$$

Then the time needed to siphon 4 L of gasoline becomes

$$\Delta t = \frac{V}{\dot{V}} = \frac{4\ \text{L}}{0.0753\ \text{L/s}} = \mathbf{53.1\ s}$$

(*b*) The pressure at point 3 can be determined by writing the Bernoulli equation between points 2 and 3. Noting that $\mathcal{V}_2 = \mathcal{V}_3$ (conservation of mass), $z_2 = 0$, and $P_2 = P_{atm}$,

$$\frac{P_2}{\rho g} + \frac{\mathcal{V}_2^2}{2g} + z_2^{\nearrow 0} = \frac{P_3}{\rho g} + \frac{\mathcal{V}_3^2}{2g} + z_3 \quad \rightarrow \quad \frac{P_{atm}}{\rho g} = \frac{P_3}{\rho g} + z_3$$

Solving for P_3 and substituting,

$$\begin{aligned} P_3 &= P_{atm} - \rho g z_3 \\ &= 101.3\ \text{kPa} - (750\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2.75\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right) \\ &= \mathbf{81.1\ kPa} \end{aligned}$$

Discussion The siphoning time is determined by neglecting frictional effects, and thus this is the *minimum time* required. In reality, the time will be longer than 53.1 s because of the friction between the gasoline and the tube surface. Also, the pressure at point 3 is below the atmospheric pressure. If the elevation difference between points 1 and 3 is too high, the pressure at point 3 may drop below the vapor pressure of gasoline at the gasoline temperature, and some gasoline may evaporate. The vapor then may form a pocket at the top and halt the flow of gasoline.

EXAMPLE 12–6 Velocity Measurement by a Pitot Tube

A piezometer and a pitot tube are tapped into a horizontal water pipe, as shown in Fig. 12–27, to measure static and stagnation (static + dynamic) pressures. For the indicated water column heights, determine the velocity at the center of the pipe.

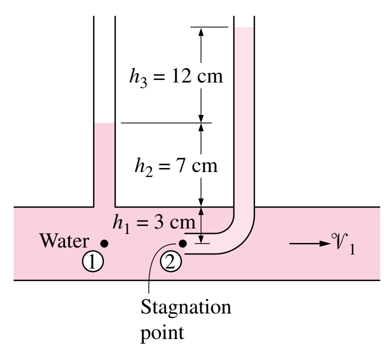


FIGURE 12–27
Schematic for Example 12–6.

SOLUTION The static and stagnation pressures in a horizontal pipe are measured. The velocity at the center of the pipe is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** Points 1 and 2 are close enough together that the irreversible energy loss between these two points is negligible, and thus we can use the Bernoulli equation.

Analysis We take points 1 and 2 along the centerline of the pipe, with point 1 directly under the piezometer and point 2 at the tip of the pitot tube. This is a steady flow with straight and parallel streamlines, and the gage pressures at points 1 and 2 can be expressed as

$$P_1 = \rho g(h_1 + h_2)$$
$$P_2 = \rho g(h_1 + h_2 + h_3)$$

Noting that point 2 is a stagnation point and thus $\mathcal{V}_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g} + \frac{\mathcal{V}_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{\mathcal{V}_2^2}{2g}^{\nearrow 0} + z_2 \quad \rightarrow \quad \frac{\mathcal{V}_1^2}{2g} = \frac{P_2 - P_1}{\rho g}$$

Substituting the P_1 and P_2 expressions gives

$$\frac{\mathcal{V}_1^2}{2g} = \frac{P_2 - P_1}{\rho g} = \frac{\rho g(h_1 + h_2 + h_3) - \rho g(h_1 + h_2)}{\rho g} = h_3$$

Solving for $\mathcal{V}_1$ and substituting,

$$\mathcal{V}_1 = \sqrt{2gh_3} = \sqrt{2(9.81 \text{ m/s}^2)(0.12 \text{ m})} = \mathbf{1.53 \text{ m/s}}$$

Discussion Note that to determine the flow velocity, all we need is to measure the height of the excess fluid column in the pitot tube.

EXAMPLE 12–7 The Rise of the Ocean Due to Hurricanes

A hurricane is a tropical storm formed over the ocean by low atmospheric pressures. As a hurricane approaches land, inordinate ocean swells (very high tides) accompany the hurricane. A Class-5 hurricane features winds in excess of 155 mph, although the wind velocity at the center "eye" is very low.

Figure 12–28 depicts a hurricane hovering over the ocean swell below. The atmospheric pressure 200 miles from the eye is 30.0 inHg (at point 1, generally normal for the ocean) and the winds are calm. The hurricane atmospheric pressure at the eye of the storm is 22.0 inHg. Estimate the ocean swell at (*a*) the eye of the hurricane at point 3 and (*b*) point 2, where the wind velocity is 155 mph. Take the density of seawater and mercury to be 64 lbm/ft^3 and 848 lbm/ft^3, respectively, and the density of air at normal sea level temperature and pressure to be 0.076 lbm/ft^3.

SOLUTION A hurricane is moving over the ocean. The amount of ocean swell at the eye and at active regions of the hurricane are to be determined.

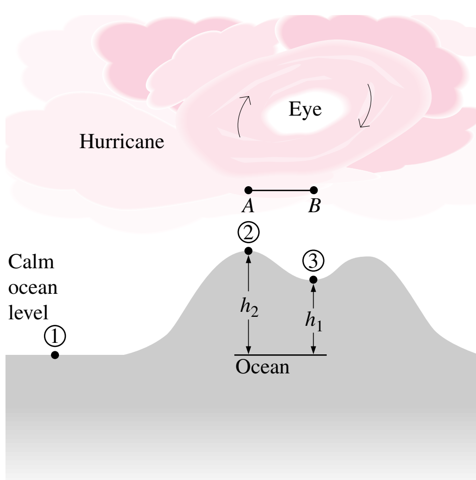


FIGURE 12–28
Schematic for Example 12–7. The vertical scale is exaggerated.

Assumptions **1** The airflow within the hurricane is steady, incompressible, and irrotational (so that the Bernoulli equation is applicable). (This is certainly a very questionable assumption for a highly turbulent flow, but it is justified in the solution.) **2** The effect of water drifted into the air is negligible.

Properties The densities of air at normal conditions, seawater, and mercury are given to be 0.076 lbm/ft^3, 64 lbm/ft^3, and 848 lbm/ft^3, respectively.

Analysis (*a*) Reduced atmospheric pressure over the water causes the water to rise. Thus, decreased pressure at point 2 relative to point 1 causes the ocean water to rise at point 2. The same is true at point 3, where the storm air velocity is negligible. The pressure difference given in terms of the mercury column height can be expressed in terms of the seawater column height by

$$\Delta P = (\rho g h)_{\text{Hg}} = (\rho g h)_{\text{sw}} \qquad \rightarrow \qquad h_{\text{sw}} = \frac{\rho_{\text{Hg}}}{\rho_{\text{sw}}} h_{\text{Hg}}$$

Then the pressure difference between points 1 and 2 in terms of the seawater column height becomes

$$h_1 = \frac{\rho_{\text{Hg}}}{\rho_{\text{sw}}} h_{\text{Hg}} = \left(\frac{848 \text{ lbm/ft}^3}{64 \text{ lbm/ft}^3}\right)[(30 - 22) \text{ inHg}]\left(\frac{1 \text{ ft}}{12 \text{ in}}\right) = \mathbf{8.83 \text{ ft}}$$

which is equivalent to the storm surge at the *eye of the hurricane* since the wind velocity there is negligible and there are no dynamic effects.

(*b*) To determine the additional rise of ocean water at point 2 due to the high winds at that point, we write the Bernoulli equation between points *A* and *B*, which are on top of the points 2 and 3, respectively. Noting that $\mathcal{V}_B \cong 0$ (the eye region of the hurricane is relatively calm) and $z_A = z_B$ (both points are on the same horizontal line), the Bernoulli equation simplifies to

$$\frac{P_A}{\rho g} + \frac{\mathcal{V}_A^2}{2g} + \cancel{z_A} = \frac{P_B}{\rho g} + \overbrace{\frac{\mathcal{V}_B^2}{2g}}^{0} + \cancel{z_B} \qquad \rightarrow \qquad \frac{P_B - P_A}{\rho g} = \frac{\mathcal{V}_A^2}{2g}$$

Substituting,

$$\frac{P_B - P_A}{\rho g} = \frac{\mathcal{V}_A^2}{2g} = \frac{(155 \text{ mph})^2}{2(32.2 \text{ ft/s}^2)}\left(\frac{1.4667 \text{ ft/s}}{1 \text{ mph}}\right)^2 = 803 \text{ ft}$$

where ρ is the density of air in the hurricane. Noting that the density of an ideal gas at constant temperature is proportional to absolute pressure and the density of air at the normal atmospheric pressure of 14.7 psia $\cong$ 30 inHg is 0.076 lbm/ft^3, the density of air in the hurricane is

$$\rho_{\text{air}} = \frac{P_{\text{air}}}{P_{\text{atm air}}} \rho_{\text{atm air}} = \left(\frac{22 \text{ inHg}}{30 \text{ inHg}}\right)(0.076 \text{ lbm/ft}^3) = 0.056 \text{ lbm/ft}^3$$

Using the relation developed above in part (*a*), the seawater column height equivalent to 803 ft of air column height is determined to be

$$h_{\text{dynamic}} = \frac{\rho_{\text{air}}}{\rho_{\text{sw}}} h_{\text{air}} = \left(\frac{0.056 \text{ lbm/ft}^3}{64 \text{ lbm/ft}^3}\right)(803 \text{ ft}) = 0.70 \text{ ft}$$

Therefore, the pressure at point 2 is 0.70 ft seawater column lower than the pressure at point 3 due to the high wind velocities, causing the ocean to rise an additional 0.70 ft. Then the total storm surge at point 2 becomes

$$h_2 = h_1 + h_{\text{dynamic}} = 8.83 + 0.70 = \mathbf{9.53 \text{ ft}}$$

Discussion This problem involves highly turbulent flow and the intense breakdown of the streamlines, and thus the applicability of the Bernoulli equation in part (*b*) is questionable. The Bernoulli analysis can be thought of as the limiting, ideal case, and shows that the rise of seawater due to high-velocity winds cannot be more than 0.70 ft.

The wind power of hurricanes is not the only cause of damage to coastal areas. Ocean flooding and erosion from excessive tides is just as serious, as are high waves generated by the storm turbulence and energy.

EXAMPLE 12–8 Bernoulli Equation for Compressible Flow

Derive the Bernoulli equation when the compressibility effects are not negligible for an ideal gas undergoing (*a*) an isothermal process and (*b*) an isentropic process.

SOLUTION The Bernoulli equation for compressible flow is to be obtained for an ideal gas for isothermal and isentropic processes.

Assumptions **1** The flow is steady, and frictional effects are negligible. **2** The fluid is an ideal gas, so the relation $P = \rho RT$ is applicable. **3** The specific heats are constant so that $P/\rho^k =$ *constant* during an isentropic process.

Analysis (*a*) When the compressibility effects are significant and the flow cannot be assumed to be incompressible, the Bernoulli equation is given by Eq. 12–17 as

$$\int \frac{dP}{\rho} + \frac{\mathcal{V}^2}{2} + gz = \text{constant} \qquad \textbf{(1)}$$

The compressibility effects can be properly accounted for by expressing ρ in terms of pressure, and then performing the integration $\int dP/\rho$ in Eq. (1). But this requires a relation between P and ρ for the process. For the *isothermal* expansion or compression of an ideal gas, the integral in Eq. (1) can be performed easily by noting that $T =$ constant and substituting $\rho = P/RT$. It gives

$$\int \frac{dP}{\rho} = \int \frac{dP}{P/RT} = RT \ln P$$

Substituting into Eq. (1) gives the desired relation,

Isothermal process:
$$RT \ln P + \frac{\mathcal{V}^2}{2} + gz = \text{constant} \qquad \textbf{(2)}$$

(*b*) A more practical case of compressible flow is the *isentropic flow of ideal gases* through equipment that involves high-speed fluid flow such as nozzles, diffusers, and the passages between turbine blades. Isentropic (i.e., reversible and adiabatic) flow is closely approximated by these devices, and it is characterized by the relation $P/\rho^k = C =$ *constant* where k is the specific heat ratio of the gas. Solving for ρ from $P/\rho^k = C$ gives $\rho = C^{-1/k} P^{1/k}$. Performing the integration,

$$\int \frac{dP}{\rho} = \int C^{1/k} P^{-1/k}\, dP = C^{1/k} \frac{P^{-1/k+1}}{-1/k+1} = \frac{P^{1/k}}{\rho} \frac{P^{-1/k+1}}{-1/k+1} = \left(\frac{k}{k-1}\right)\frac{P}{\rho} \qquad \textbf{(3)}$$

Substituting, the Bernoulli equation for steady, isentropic, compressible flow of an ideal gas becomes

Isentropic flow:
$$\left(\frac{k}{k-1}\right)\frac{P}{\rho} + \frac{\mathcal{V}^2}{2} + gz = \text{constant} \qquad \textbf{(4a)}$$

or

$$\left(\frac{k}{k-1}\right)\frac{P_1}{\rho_1} + \frac{\mathcal{V}_1^2}{2} + gz_1 = \left(\frac{k}{k-1}\right)\frac{P_2}{\rho_2} + \frac{\mathcal{V}_2^2}{2} + gz_2 \qquad \textbf{(4b)}$$

A common practical situation involves the acceleration of a gas from rest (stagnation conditions at state 1) with negligible change in elevation. In that case we have $z_1 = z_2$, $\mathcal{V}_1 = 0$. Noting that $\rho = P/RT$ for ideal gases, $P/\rho^k = constant$ for isentropic flow, and the Mach number is defined as $\text{Ma} = \mathcal{V}/c$ where $c = \sqrt{kRT}$ is the local speed of sound for ideal gases, the relation above simplifies to

$$\frac{P_1}{P_2} = \left[1 + \left(\frac{k-1}{2}\right)\text{Ma}_2^2\right]^{k/(k-1)} \qquad \textbf{(4c)}$$

where state 1 is the stagnation state and state 2 is any state along the flow.

Discussion It can be shown that the results obtained using the compressible and incompressible equations deviate no more than 2 percent when the Mach number is less than 0.3. Therefore, the flow of an ideal gas can be considered to be incompressible when Ma $\leq$ 0.3. For atmospheric air at normal conditions, this corresponds to a flow speed of about 100 m/s or 360 km/h, which covers our range of interest.

12–4 ▪ ENERGY ANALYSIS OF STEADY-FLOW SYSTEMS

For steady-flow systems, the time rate of change of the energy content of the control volume is zero. When the work associated with electric, magnetic, surface tension, and viscous effects are negligible, the steady-flow energy equation can be expressed as (see Chap. 5)

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \sum_{\text{out}} \dot{m}\left(h + \frac{\mathcal{V}^2}{2} + gz\right) - \sum_{\text{in}} \dot{m}\left(h + \frac{\mathcal{V}^2}{2} + gz\right) \qquad \textbf{(12–26)}$$

It states that *the net rate of energy transfer to a control volume by heat and work transfers during steady flow is equal to the difference between the rates of outgoing and incoming energy flows with mass.*

Many practical problems involve just one inlet and one outlet (Fig. 12–29). The mass flow rate for such **single-stream systems** remains constant, and Eq. 12–26 reduces to

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \dot{m}\left(h_2 - h_1 + \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} + g(z_2 - z_1)\right) \qquad \textbf{(12–27)}$$

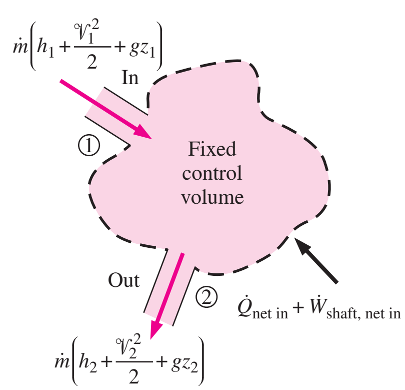


FIGURE 12–29
A control volume with only one inlet and one outlet and energy interactions.

where subscripts 1 and 2 stand for inlet and outlet, respectively. The steady-flow energy equation on a unit mass basis is obtained by dividing Eq. 12–27 by the mass flow rate $\dot{m}$,

$$q_{\text{net in}} + w_{\text{shaft, net in}} = h_2 - h_1 + \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} + g(z_2 - z_1) \qquad \textbf{(12–28)}$$

where $q_{\text{net in}} = \dot{Q}_{\text{net in}}/\dot{m}$ is the net heat transfer to the fluid per unit mass and $w_{\text{shaft, net in}} = \dot{W}_{\text{shaft, net in}}/\dot{m}$ is the work input to the fluid per unit mass. Using the definition of enthalpy $h = u + P/\rho$ and rearranging, the steady-flow energy equation can also be expressed as

$$w_{\text{shaft, net in}} + \frac{P_1}{\rho_1} + \frac{\mathcal{V}_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{\mathcal{V}_2^2}{2} + gz_2 + (u_2 - u_1 - q_{\text{net in}}) \qquad \textbf{(12–29)}$$

where u is the *internal energy*, P/ρ is the *flow energy*, $\mathcal{V}^2/2$ is the *kinetic energy*, and gz is the *potential energy* of the fluid per unit mass. These relations are valid for both compressible or incompressible flows.

The left side of Eq. 12–29 represents the mechanical energy input while the first three terms on the right side represent the mechanical energy output. If the flow is ideal with no irreversibilities such as friction, the total mechanical energy must be conserved, and the terms in parentheses $(u_2 - u_1 - q_{\text{net in}})$ must equal zero. That is,

Ideal flow (no mechanical energy loss) $$q_{\text{net in}} = u_2 - u_1 \qquad \textbf{(12–30)}$$

Any increase in $u_2 - u_1$ above $q_{\text{net in}}$ is due to the irreversible conversion of mechanical energy to thermal energy, and thus $u_2 - u_1 - q_{\text{net in}}$ represents the mechanical energy loss (Fig. 12–30). That is,

Mechanical energy loss: $$e_{\text{mech, loss}} = u_2 - u_1 - q_{\text{net in}} \qquad \textbf{(12–31)}$$

For single-phase fluids (a gas or a liquid), we have $u_2 - u_1 = C_v(T_2 - T_1)$ where C_v is the constant-volume specific heat. Then the steady-flow energy equation on a unit mass basis can be written conveniently as a **mechanical energy** balance as

$$e_{\text{mech, in}} = e_{\text{mech, out}} + e_{\text{mech, loss}} \qquad \textbf{(12–32)}$$

or

$$w_{\text{shaft, net in}} + \frac{P_1}{\rho_1} + \frac{\mathcal{V}_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{\mathcal{V}_2^2}{2} + gz_2 + e_{\text{mech, loss}} \qquad \textbf{(12–33)}$$

Noting that $w_{\text{shaft, net in}} = w_{\text{shaft, in}} - w_{\text{shaft, out}} = w_{\text{pump}} - w_{\text{turbine}}$, the mechanical energy balance can be written more explicitly as

$$\frac{P_1}{\rho_1} + \frac{\mathcal{V}_1^2}{2} + gz_1 + w_{\text{pump}} = \frac{P_2}{\rho_2} + \frac{\mathcal{V}_2^2}{2} + gz_2 + w_{\text{turbine}} + e_{\text{mech, loss}} \qquad \textbf{(12–34)}$$

where w_{pump} is the mechanical work input (due to the presence of a pump, fan, compressor, etc.) and w_{turbine} is the mechanical work output. When the flow is incompressible, either absolute or gage pressure can be used for P since P_{atm}/ρ would appear on both sides, and would cancel out.

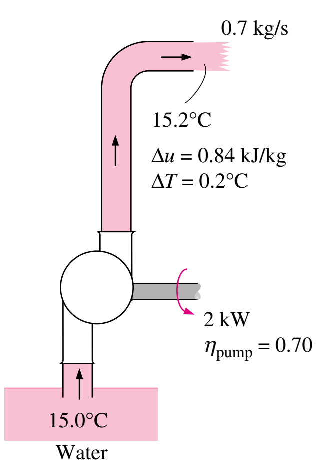
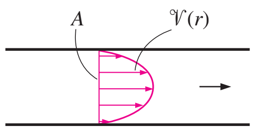


FIGURE 12–30
The lost mechanical energy in a fluid flow system results in an increase in the internal energy of the fluid, and thus in a rise of fluid temperature.

Kinetic Energy Correction Factor, α

The mean flow velocity $\mathcal{V}_m$ was defined such that the relation $\rho \mathcal{V}_m A$ gives the actual mass flow rate. Therefore, there is no such thing as a correction factor for mass flow rate. However, the kinetic energy of a fluid stream obtained from $\mathcal{V}^2/2$ is not the same as the actual kinetic energy of the fluid stream since the square of a sum is not equal to the sum of the squares of its components (Fig. 12–32). This error can be corrected by replacing the kinetic energy terms $\mathcal{V}^2/2$ in the energy equation by $\alpha \mathcal{V}_m^2/2$ where α is the **kinetic energy correction factor.** By using equations for the variation of velocity with the radial distance, it can be shown that the correction factor is 2.0 for fully developed laminar pipe flow, and it ranges between 1.04 and 1.11 for turbulent flow in a circular pipe.

The kinetic energy correction factors are usually disregarded in an elementary analysis since (1) most flows encountered in practice are turbulent, for which the correction factor is near unity, and (2) the kinetic energy terms are usually small relative to the other terms in the energy equation, and multiplying them by a factor less than 2.0 does not make much difference. Besides, when the velocity and thus the kinetic energy are high, the flow turns turbulent. Therefore, we will not consider the kinetic energy correction factor in the analysis. However, the reader should keep in mind that he or she may encounter situations for which these factors are significant, especially when the flow is laminar.



$$\dot m = \rho \mathcal{V}_m A, \rho = \text{constant}$$

$$\dot{KE}_{act} = \int ke\, \delta \dot m = \int_A \frac{1}{2} \mathcal{V}^2(r)[\rho \mathcal{V}(r)\, dA] = \frac{1}{2} \rho \int_A \mathcal{V}^3(r)\, dA$$

$$\dot{KE}_m = \frac{1}{2} \dot m \mathcal{V}_m^2 = \frac{1}{2} \rho A \mathcal{V}_m^3$$

$$\alpha = \frac{\dot{KE}_{act}}{\dot{KE}_m} = \frac{1}{A} \int_A \left(\frac{\mathcal{V}(r)}{\mathcal{V}_m}\right)^3 dA$$

FIGURE 12–32
The determination of the *kinetic energy correction factor* using actual velocity distribution $\mathcal{V}(r)$ and the mean velocity $\mathcal{V}_m$ at a cross section.

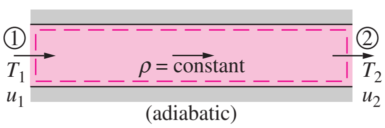


FIGURE 12–33
Schematic for Example 12–9.

EXAMPLE 12–9 Effect of Friction on Fluid Temperature and Head Loss

Show that during steady and incompressible flow of a fluid in an adiabatic flow section (*a*) the temperature remains constant and there is no head loss when the flow is frictionless and (*b*) the temperature increases and some head loss occurs when there are frictional effects. Discuss if it is possible for the fluid temperature to decrease during such flow (Fig. 12–33).

SOLUTION Steady and incompressible flow through an adiabatic section is considered. The effects of friction on the temperature and the heat loss are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The flow section is adiabatic and thus there is no heat transfer.

Analysis The density of a fluid remains constant during incompressible flow, and the entropy change of an incompressible system is

$$\Delta s = C_v \ln \frac{T_2}{T_1}$$

This relation represents the entropy change of the fluid per unit mass as it flows through the flow section from state 1 at the inlet to state 2 at the exit. Entropy change is caused by two effects: (1) heat transfer and (2) irreversibilities. Therefore, in the absence of heat transfer, entropy change is due to irreversibilities only whose effect is always to increase entropy.

(*a*) The entropy change of the fluid is zero when the process does not involve any irreversibilities such as friction and swirling, and thus for reversible flow we have

Temperature change: $\Delta s = C_v \ln \frac{T_2}{T_1} = 0 \quad \rightarrow \quad T_2 = T_1$

Mechanical energy loss: $e_{\text{mech, loss}} = u_2 - u_1 = C_v(T_2 - T_1) = 0$

Head loss: $h_L = e_{\text{mech, loss}}/g = 0$

Thus we conclude that when heat transfer and frictional effects are negligible, (1) the temperature of the fluid remains constant, (2) no mechanical energy is converted to thermal energy, and (3) there is no head loss.

(*b*) When there are irreversibilities such as friction, the entropy change is positive and thus we have:

Temperature change: $\Delta s = C_v \ln \frac{T_2}{T_1} > 0 \quad \rightarrow \quad T_2 > T_1$

Mechanical energy loss: $e_{\text{mech, loss}} = u_2 - u_1 = C_v(T_2 - T_1) > 0$

Head loss: $h_L = e_{\text{mech, loss}}/g > 0$

Thus we conclude that when the flow is adiabatic and irreversible, (1) the temperature of the fluid increases, (2) some mechanical energy is converted to thermal energy, and (3) some head loss occurs.

Discussion It is impossible for the fluid temperature to decrease during steady, incompressible, adiabatic flow since this would require the entropy of an adiabatic system to decrease, which would be a violation of the second law of thermodynamics.

EXAMPLE 12–10 Pumping Power and Frictional Heating in a Pump

The pump of a water distribution system is powered by a 15-kW electric motor whose efficiency is 90 percent (Fig. 12–34). The water flow rate through the pump is 50 L/s. The diameters of the inlet and exit pipes are the same, and the elevation difference across the pump is negligible. If the pressures at the inlet and outlet of the pump are measured to be 100 kPa and 300 kPa (absolute), respectively, determine (*a*) the mechanical efficiency of the pump and (*b*) the temperature rise of water as it flows through the pump due to the mechanical inefficiency.

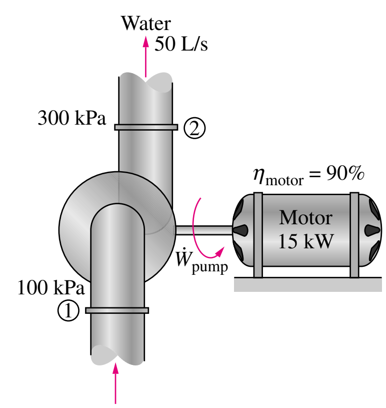


FIGURE 12–34
Schematic for Example 12–10.

SOLUTION The pressures across a pump are measured. The mechanical efficiency of the pump and the temperature rise of water are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The pump is driven by an external motor so that the heat generated by the motor is dissipated to the atmosphere. **3** The elevation difference between the inlet and outlet of the pump is negligible, $z_1 \cong z_2$. **4** The inlet and outlet diameters are the same and thus the inlet and exit velocities are equal, $\mathcal{V}_1 = \mathcal{V}_2$.

Properties We take the density of water to be 1 kg/L = 1000 kg/m^3 and its specific heat to be 4.18 kJ/kg · °C.

Analysis (*a*) The mass flow rate of water through the pump is

$$\dot{m} = \rho \dot{V} = (1 \text{ kg/L})(50 \text{ L/s}) = 50 \text{ kg/s}$$

The motor draws 15 kW of power and is 90 percent efficient. Thus the mechanical (shaft) power it delivers to the pump is

$$\dot{W}_{\text{pump, shaft}} = \eta_{\text{motor}} \dot{W}_{\text{electric}} = (0.90)(15 \text{ kW}) = 13.5\text{kW}$$

To determine the mechanical efficiency of the pump, we need to know the increase in the mechanical energy of the fluid as it flows through the pump, which is

$$\Delta \dot{E}_{\text{mech, fluid}} = \dot{E}_{\text{mech, out}} - \dot{E}_{\text{mech, in}} = \dot{m}\left(\frac{P_2}{\rho} + \frac{\mathcal{V}_2^2}{2} + gz_2\right) - \dot{m}\left(\frac{P_1}{\rho} + \frac{\mathcal{V}_1^2}{2} + gz_1\right)$$

Simplifying it for this case and substituting the given values,

$$\Delta \dot{E}_{\text{mech, fluid}} = \dot{m}\left(\frac{P_2 - P_1}{\rho}\right) = (50 \text{ kg/s})\left(\frac{(300 - 100) \text{ kPa}}{1000 \text{ kg/m}^3}\right)\left(\frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3}\right) = 10 \text{ kW}$$

Then the mechanical efficiency of the pump becomes

$$\eta_{\text{pump}} = \frac{\Delta \dot{E}_{\text{mech, fluid}}}{\dot{W}_{\text{pump, shaft}}} = \frac{10 \text{ kW}}{13.5 \text{kW}} = \mathbf{0.741} \quad \text{or} \quad \mathbf{74.1\%}$$

(*b*) Of the 13.5-kW mechanical power supplied by the pump, only 10 kW is imparted to the fluid as mechanical energy. The remaining 3.5 kW is converted to thermal energy due to frictional effects, and this "lost" mechanical energy manifests itself as a heating effect in the fluid,

$$\dot{E}_{\text{mech, loss}} = \dot{W}_{\text{pump, shaft}} - \Delta \dot{E}_{\text{mech, fluid}} = 13.5 - 10 = 3.5 \text{ kW}$$

The temperature rise of water due to this mechanical inefficiency is determined from the thermal energy balance, $\dot{E}_{\text{mech, loss}} = \dot{m}(u_2 - u_1) = \dot{m}C\Delta T$. Solving for ΔT,

$$\Delta T = \frac{\dot{E}_{\text{mech, loss}}}{\dot{m}C} = \frac{3.5 \text{ kW}}{(50 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot {}^\circ\text{C})} = \mathbf{0.017^\circ C}$$

Therefore, the water will experience a temperature rise of 0.017°C, which is very small, as it flows through the pump.

Discussion In an actual application, the temperature rise of water will probably be less since part of the heat generated will be transferred to the casing of the pump, and from the casing to the surrounding air. If the entire pump motor were submerged in water, then the 1.5 kW dissipated to the air due to motor inefficiency would also be transferred to the surrounding water as heat. This would cause the water temperature to rise more.

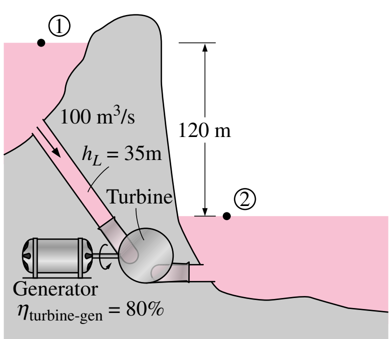


FIGURE 12–35
Schematic for Example 12–11.

EXAMPLE 12–11 Hydroelectric Power Generation from a Dam

In a hydroelectric power plant, 100 m^3/s of water flows from an elevation of 120 m to a turbine, where electric power is generated (Fig. 12–35). The total head loss in the system from point 1 to point 2 (excluding the turbine unit) is determined to be 35 m. If the overall efficiency of the turbine-generator is 80 percent, estimate the electrical power output.

SOLUTION The available head, flow rate, head loss, and efficiency of a hydroelectric turbine are given. The electrical power output is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The water levels at the reservoir and the discharge site remain constant.

Properties We take the density of water to be 1000 kg/m^3.
Analysis The mass flow rate of water through the turbine is

$$\dot{m} = \rho \dot{V} = (1000 \text{ kg/m}^3)(100 \text{ m}^3\text{/s}) = 10^5 \text{ kg/s}$$

We take point 2 as the reference level, and thus $z_2 = 0$. Also, both points 1 and 2 are open to the atmosphere ($P_1 = P_2 = P_{atm}$) and the flow velocities are negligible at both points ($\mathcal{V}_1 = \mathcal{V}_2 = 0$). Then the energy equation for steady incompressible flow reduces to

$$\frac{P_1}{\rho g} + \frac{\mathcal{V}_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \frac{\mathcal{V}_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L \quad \rightarrow \quad h_{\text{turbine, e}} = z_1 - h_L$$

Substituting, the extracted turbine head and the corresponding turbine power are determined to be

$$h_{\text{turbine, e}} = z_1 - h_L = 120 - 35 = 85 \text{ m}$$

$$\dot{W}_{\text{turbine, e}} = \dot{m} g h_{\text{turbine, e}} = (10^5 \text{ kg/s})(9.81 \text{ m/s}^2)(85 \text{ m})\left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2\text{/s}^2}\right) = 83{,}400 \text{ kW}$$

Therefore, a perfect turbine-generator would generate 83,400 kW of electricity from this resource. The electric power generated by the actual unit is

$$\dot{W}_{\text{electric}} = \eta_{\text{turbine-gen}} \dot{W}_{\text{turbine, e}} = (0.80)(83.4 \text{ MW}) = \mathbf{66.7 \text{ MW}}$$

Discussion Note that the power generation will increase by almost 1 MW for each percentage point improvement in the efficiency of the turbine-generator unit.

EXAMPLE 12–12 Fan Selection for Air Cooling of a Computer

A fan is to be selected to cool a computer case whose dimensions are 12 cm × 40 cm × 40 cm (Fig. 12–36). Half of the volume in the case is expected to be filled with components and the other half to be air space. A 6-cm-diameter hole is available at the front of the case for the installation of the fan that is to replace the air in the void spaces of the case once every second. Small low-power fan-motor-combined units are available in the market and their efficiency is estimated to be 30 percent. Determine (*a*) the wattage of the fan-motor unit to be purchased and (*b*) the pressure difference across the fan. Take the air density to be 1.20 kg/m^3.

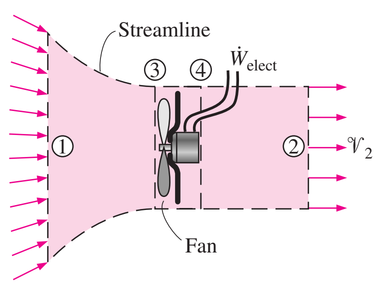


FIGURE 12–36
Schematic for Example 12–12.

SOLUTION A fan is to cool a computer case by completely replacing the air inside once every second. The power of the fan and the pressure difference across it are to be determined.
Assumptions **1** The flow is steady and incompressible. **2** Losses other than those due to the inefficiency of the fan-motor unit are negligible ($h_L = 0$).
Properties The density of air is given to be 1.20 kg/m^3.
Analysis (*a*) Noting that half of the volume of the case is occupied by the components, the air volume in the computer case is

$$V = \text{(Void fraction)(Total case volume)}$$
$$= 0.5(12 \text{ cm} \times 40 \text{ cm} \times 40 \text{ cm}) = 9600 \text{ cm}^3$$

Therefore, the volume and mass flow rates of air through the case are

$$\dot{V} = \frac{V}{\Delta t} = \frac{9600\ \text{cm}^3}{1\ \text{s}} = 9600\ \text{cm}^3/\text{s} = 9.6 \times 10^{-3}\ \text{m}^3/\text{s}$$

$$\dot{m} = \rho \dot{V} = (1.20\ \text{kg/m}^3)(9.6 \times 10^{-3}\ \text{m}^3/\text{s}) = 0.0115\ \text{kg/s}$$

The cross-sectional area of the opening in the case and the average air velocity are

$$A = \frac{\pi D^2}{4} = \frac{\pi (0.06\ \text{m})^2}{14} = 2.83 \times 10^{-3}\ \text{m}^2$$

$$\mathcal{V} = \frac{\dot{V}}{A} = \frac{9.6 \times 10^{-3}\ \text{m}^3/\text{s}}{2.83 \times 10^{-3}\ \text{m}^2} = 3.39\ \text{m/s}$$

We draw the control volume around the fan such that both the inlet and the exit are at the atmospheric pressure ($P_1 = P_2 = P_{atm}$), as shown in the figure, and the inlet section 1 is large and far from the fan so that the flow velocity at the inlet section is negligible ($\mathcal{V}_1 \cong 0$). Noting that $z_1 = z_2$ and the mechanical losses due to fan inefficiency and the frictional effects in flow are disregarded, the energy equation simplifies to

$$\dot{m}\left(\frac{P_1}{\rho} + \frac{\mathcal{V}_1^2}{2}^{\nearrow 0} + gz_1^{\nearrow 0}\right) + \dot{W}_{\text{fan}} = \dot{m}\left(\frac{P_2}{\rho} + \frac{\mathcal{V}_2^2}{2} + gz_2^{\nearrow 0}\right) + \dot{W}_{\text{turbine}}^{\nearrow 0} + \dot{E}_{\text{mech, loss}}^{\nearrow 0}$$

Solving for $\dot{W}_{\text{fan}}$ and substituting,

$$\dot{W}_{\text{fan}} = \dot{m}\frac{\mathcal{V}_2^2}{2} = (0.0115\ \text{kg/s})\frac{(3.39\ \text{m/s})^2}{2}\left(\frac{1\ \text{N}}{1\ \text{kg} \cdot \text{m/s}^2}\right) = 0.066\ \text{W}$$

Then the required electrical power input to the fan is determined to be

$$\dot{W}_{\text{elect}} = \frac{\dot{W}_{\text{fan}}}{\eta_{\text{fan-motor}}} = \frac{0.066\ \text{W}}{0.3} = \mathbf{0.22\ W}$$

Therefore, a fan-motor rated at 0.22 W is adequate for this job.

(*b*) To determine the pressure difference across the fan unit, we take points 3 and 4 to be on the two sides of the fan on a horizontal line. This time again $z_3 = z_4$ and $\mathcal{V}_3 = \mathcal{V}_4$ since the fan is a narrow cross section. Disregarding mechanical losses, the energy equation reduces to

$$\dot{m}\frac{P_3}{\rho} + \dot{W}_{\text{fan}} = \dot{m}\frac{P_4}{\rho} + \dot{E}_{\text{mech, loss}}^{\nearrow 0} \quad \rightarrow \quad \dot{W}_{\text{fan}} = \dot{m}\frac{P_4 - P_3}{\rho}$$

Solving for $P_4 - P_3$ and substituting,

$$P_4 - P_3 = \frac{\rho \dot{W}_{\text{fan}}}{\dot{m}} = \frac{(1.2\ \text{kg/m}^3)(0.066\ \text{W})}{0.0115\ \text{kg/s}}\left(\frac{1\ \text{Pa} \cdot \text{m}^3}{1\ \text{Ws}}\right) = \mathbf{6.9\ Pa}$$

Therefore, the pressure rise across the fan is 6.9 Pa.

Discussion The efficiency of the fan-motor unit is given to be 30 percent, which means 30 percent of the electric power $\dot{W}_{\text{electric}}$ consumed by the unit will be converted to useful mechanical energy while the rest (70 percent) will be "lost" and converted to thermal energy. Also, a much larger fan is required in an actual system to overcome frictional losses inside the computer case.

EXAMPLE 12–13 Head and Power Loss during Water Pumping

Water is pumped from a lower reservoir to a higher reservoir by a pump that provides 20 kW of useful mechanical power to water (Fig. 12–37). The free surface of the upper reservoir is 45 m higher than the surface of the lower reservoir. If the flow rate of water is measured to be 0.03 m^3/s, determine the head loss of the system and the lost mechanical power during this process.

SOLUTION Water is pumped from a lower reservoir to a higher one. The head and power loss associated with this process are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The elevation difference between the reservoirs is constant.

Properties We take the density of water to be 1000 kg/m^3.

Analysis The mass flow rate of water through the system is

$$\dot{m} = \rho \dot{V} = (1000\ \text{kg/m}^3)(0.03\ \text{m}^3\text{/s}) = 30\ \text{kg/s}$$

We choose points 1 and 2 at the free surfaces of the lower and upper reservoirs, respectively, and take the surface of the lower reservoir as the reference level ($z_1 = 0$). Both points are open to the atmosphere ($P_1 = P_2 = P_{atm}$) and the velocities at both locations are negligible ($\mathcal{V}_1 = \mathcal{V}_2 = 0$). Then the energy equation for steady incompressible flow for a control volume between 1 and 2 reduces to

$$\dot{m}\left(\frac{P_1}{\rho} + \frac{\mathcal{V}_1^2}{2}\overset{0}{\nearrow} + gz_1\overset{0}{\nearrow}\right) + \dot{W}_{\text{pump}} = \dot{m}\left(\frac{P_2}{\rho} + \frac{\mathcal{V}_2^2}{2}\overset{0}{\nearrow} + gz_2\right) + \dot{W}_{\text{turbine}}\overset{0}{\nearrow} + \dot{E}_{\text{mech, loss}}$$

$$\dot{W}_{\text{pump}} = \dot{m}gz_2 + \dot{E}_{\text{mech, loss}} \quad \rightarrow \quad \dot{E}_{\text{mech, loss}} = \dot{W}_{\text{pump}} - \dot{m}gz_2$$

Substituting, the lost mechanical power and head loss are determined to be

$$\dot{E}_{\text{mech, loss}} = 20\ \text{kW} - (30\ \text{kg/s})(9.81\ \text{m/s}^2)(45\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kW}}{1000\ \text{N}\cdot\text{m/s}}\right)$$
$$= \mathbf{6.76\ kW}$$

$$h_L = \frac{\dot{E}_{\text{mech, loss}}}{\dot{m}g} = \frac{6.76\ \text{kW}}{(30\ \text{kg/s})(9.81\ \text{m/s}^2)}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)\left(\frac{1000\ \text{N}\cdot\text{m/s}}{1\ \text{kW}}\right) = \mathbf{23.0\ m}$$

since the entire mechanical losses are due to frictional losses in the pipes.

Discussion The 6.76 kW of power is used to overcome the friction in the piping system. Note that the pump could raise the water an additional 23 m if there were no losses in the system. In this ideal case, the pump would function as a turbine when the water is allowed to flow from the upper reservoir to the lower reservoir, and generate 20 kW of power.

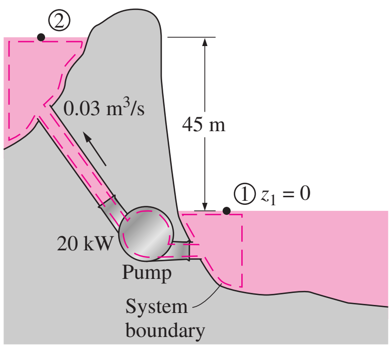


FIGURE 12–37
Schematic for Example 12–13.

SUMMARY

This chapter deals with the Bernoulli and the energy equations, and their applications. The *mechanical energy* is the form of energy associated with the velocity, elevation, and pressure of the fluid, and it can be converted to mechanical work completely and directly by an ideal mechanical device. The efficiencies of various devices are defined as

$$\eta_{\text{pump}} = \frac{\text{Mechanical energy increase of the fluid}}{\text{Mechanical energy input}} = \frac{\Delta \dot{E}_{\text{mech, fluid}}}{\dot{W}_{\text{shaft, in}}} = \frac{\dot{W}_{\text{pump, u}}}{\dot{W}_{\text{pump}}}$$

$$\eta_{\text{turbine}} = \frac{\text{Mechanical energy output}}{\text{Mechanical energy decrease of the fluid}} = \frac{\dot{W}_{\text{shaft, out}}}{|\Delta \dot{E}_{\text{mech, fluid}}|} = \frac{\dot{W}_{\text{turbine}}}{\dot{W}_{\text{turbine, e}}}$$

$$\eta_{\text{motor}} = \frac{\text{Mechanical power output}}{\text{Electrical power input}} = \frac{\dot{W}_{\text{shaft, out}}}{\dot{W}_{\text{elect, in}}}$$

$$\eta_{\text{generator}} = \frac{\text{Electrical power output}}{\text{Mechanical power input}} = \frac{\dot{W}_{\text{elect, out}}}{\dot{W}_{\text{shaft, in}}}$$

$$\eta_{\text{pump-motor}} = \eta_{\text{pump}}\,\eta_{\text{motor}} = \frac{\dot{E}_{\text{mech, out}} - \dot{E}_{\text{mech, in}}}{\dot{W}_{\text{elect, in}}} = \frac{\Delta \dot{E}_{\text{mech, fluid}}}{\dot{W}_{\text{elect, in}}}$$

$$\eta_{\text{turbine-gen}} = \eta_{\text{turbine}}\,\eta_{\text{generator}} = \frac{\dot{W}_{\text{elect, out}}}{\dot{E}_{\text{mech, in}} - \dot{E}_{\text{mech, out}}} = \frac{\dot{W}_{\text{elect, out}}}{|\Delta \dot{E}_{\text{mech, fluid}}|}$$

The *Bernoulli equation* is a relation between pressure, velocity, and elevation in steady incompressible flow, and is expressed along a streamline and in regions where net viscous forces are negligible as

$$\frac{P}{\rho} + \frac{\mathcal{V}^2}{2} + gz = \text{constant}$$

It can also be expressed between any two points on the streamline as

$$\frac{P_1}{\rho} + \frac{\mathcal{V}_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{\mathcal{V}_2^2}{2} + gz_2$$

The Bernoulli equation is an expression of mechanical energy balance and can be stated *as the sum of the kinetic, potential, and flow energies of a fluid particle is constant along a streamline during steady flow when the compressibility and frictional effects are negligible.* Multiplying the Bernoulli equation by density gives

$$P + \rho\frac{\mathcal{V}^2}{2} + \rho gz = \text{constant}$$

where P is the *static pressure,* which represents the actual pressure of the fluid; $\rho\mathcal{V}^2/2$ is the *dynamic pressure,* which represents the pressure rise when the fluid in motion is brought to a stop; and ρgz is the *hydrostatic pressure,* which accounts for the effects of fluid weight on pressure. The sum of the static, dynamic, and hydrostatic pressures is called the *total pressure.* The Bernoulli equation states that *the total pressure along a streamline is constant.* The sum of the static and dynamic pressures is called the *stagnation pressure,* which represents the pressure at a point where the fluid is brought to a complete stop in a frictionless manner. The Bernoulli equation can also be represented in terms of "heads" by dividing each term by g,

$$\frac{P}{\rho g} + \frac{\mathcal{V}^2}{2g} + z = H = \text{constant}$$

where $P/\rho g$ is the *pressure head,* which represents the height of a fluid column that produces the static pressure P; $\mathcal{V}^2/2g$ is the *velocity head,* which represents the elevation needed for a fluid to reach the velocity $\mathcal{V}$ during frictionless free fall; and z is the *elevation head,* which represents the potential energy of the fluid. Also, H is the *total head* for the flow. The line that represents the sum of the static pressure and the elevation heads, $P/\rho g + z$, is called the *hydraulic grade line* (HGL), and the line that represents the total head of the fluid, $P/\rho g + \mathcal{V}^2/2g + z$, is called the *energy grade line* (EGL).

The *energy equation* for steady incompressible flow can be expressed as

$$\frac{P_1}{\rho} + \frac{\mathcal{V}_1^2}{2} + gz_1 + w_{\text{pump}} = \frac{P_2}{\rho} + \frac{\mathcal{V}_2^2}{2} + gz_2 + w_{\text{turbine}} + e_{\text{mech, loss}}$$

$$\dot{m}\left(\frac{P_1}{\rho} + \frac{\mathcal{V}_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump}} = \dot{m}\left(\frac{P_2}{\rho} + \frac{\mathcal{V}_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech, loss}}$$

$$\frac{P_1}{\rho g} + \frac{\mathcal{V}_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \frac{\mathcal{V}_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L$$

where

$$h_{\text{pump, u}} = \frac{w_{\text{pump, u}}}{g} = \frac{\dot{W}_{\text{pump, u}}}{\dot{m}g}$$

$$h_{\text{turbine, e}} = \frac{w_{\text{turbine, e}}}{g} = \frac{\dot{W}_{\text{turbine, e}}}{\dot{m}g}$$

$$h_L = \frac{e_{\text{mech loss, piping}}}{g} = \frac{\dot{E}_{\text{mech loss, piping}}}{\dot{m}g}$$

$$e_{\text{mech, loss}} = u_2 - u_1 - q_{\text{net in}}$$

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PROBLEMS*

Mechanical Energy and Efficiency

12–1C What is mechanical energy? How does it differ from thermal energy? What are the forms of mechanical energy of a fluid stream?

12–2C What is mechanical efficiency? What does a mechanical efficiency of 100 percent mean for a hydraulic turbine?

12–3C How is the combined pump-motor efficiency of a pump and motor system defined? Can the combined pump-motor efficiency be greater than either of the pump or the motor efficiency?

12–4C Define turbine efficiency, generator efficiency, and combined turbine-generator efficiency.

12–5 Consider a river flowing toward a lake at an average velocity of 3 m/s at a rate of 500 m^3/s at a location 90 m above the lake surface. Determine the total mechanical energy of the river water per unit mass and the power generation potential of the entire river at that location. *Answer:* 444 MW

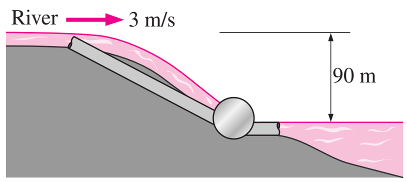

FIGURE P12–5

*Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with a CD-EES icon are solved using EES, and complete solutions together with parametric studies are included on the enclosed CD. Problems with a computer-EES icon are comprehensive in nature, and are intended to be solved with a computer, preferably using the EES software that accompanies this text.

12–6 Electrical power is to be generated by installing a hydraulic turbine-generator at a site 70 m below the free surface of a large water reservoir that can supply water at a rate of 1500 kg/s steadily. If the mechanical power output of the turbine is 800 kW and the electrical power generation is 750 kW, determine the turbine efficiency and the combined turbine-generator efficiency of this plant. Neglect losses in the pipes.

12–7 At a certain location, wind is blowing steadily at 12 m/s. Determine the mechanical energy of air per unit mass and the power generation potential of a wind turbine with 50-m-diameter blades at that location. Also determine the actual electric power generation assuming an overall efficiency of 30 percent. Take the air density to be 1.25 kg/m^3.

12–8 Reconsider Prob. 12–7. Using EES (or other) software, investigate the effect of wind velocity and the blade span diameter on wind power generation. Let the velocity vary from 5 m/s to 20 m/s in increments of 5 m/s, and the diameter to vary from 20 m to 80 m in increments of 20 m. Tabulate the results, and discuss their significance.

12–9E A differential thermocouple with sensors at the inlet and exit of a pump indicates that the temperature of water rises 0.072°F as it flows through the pump at a rate of 1.5 ft^3/s. If the shaft power input to the pump is 27 hp, determine the mechanical efficiency of the pump. *Answer:* 64.7%

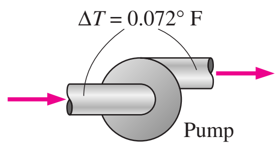

FIGURE P12–9E

12–10 Water is pumped from a lake to a storage tank 20 m above at a rate of 70 L/s while consuming 20.4 kW of electric power. Disregarding any frictional losses in the pipes and any changes in kinetic energy, determine (*a*) the overall efficiency of the pump-motor unit and (*b*) the pressure difference between the inlet and the exit of the pump.

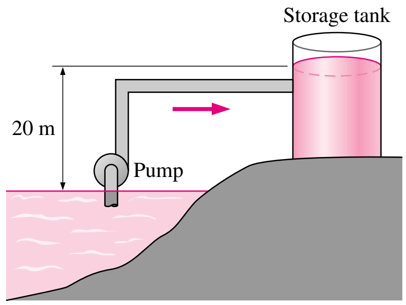

FIGURE P12–10

Bernoulli Equation

12–11C What is streamwise acceleration? How does it differ from normal acceleration? Can a fluid particle accelerate in steady flow?

12–12C Express the Bernoulli equation in three different ways using (*a*) energies, (*b*) pressures, and (*c*) heads.

12–13C What are the three major assumptions used in the derivation of the Bernoulli equation?

12–14C Define static, dynamic, and hydrostatic pressure. Under what conditions is their sum constant for a flow stream?

12–15C What is stagnation pressure? Explain how it can be measured.

12–16C Define pressure head, velocity head, and elevation head for a fluid stream and express them for a fluid stream whose pressure is P, velocity is $\mathcal{V}$, and elevation is z.

12–17C What is the hydraulic grade line? How does it differ from the energy grade line? Under what conditions do both lines coincide with the free surface of a liquid?

12–18C How is the location of the hydraulic grade line determined for open-channel flow? How is it determined at the exit of a pipe discharging to the atmosphere?

12–19C The water level of a tank on a building roof is 20 m above the ground. A hose leads from the tank bottom to the ground. The end of the hose has a nozzle, which is pointed straight up. What is the maximum height to which the water could rise? What factors would reduce this height?

12–20C In a certain application, a siphon must go over a high wall. Can water or oil with a specific gravity of 0.8 go over a higher wall? Why?

12–21C Explain how and why a siphon works. Someone proposes siphoning cold water over a 7-m-high wall. Is this feasible? Explain.

12–22C A student siphons water over a 8.5-m wall at sea level. He then climbs to the summit of Mount Shasta (elevation 4390 m, P_{atm} = 58.5 kPa) and attempts the same experiment. Comment on his prospects for success.

12–23C A glass manometer with oil as the working fluid is connected to an air duct as shown in the figure. Will the oil in the manometer move as in figure (*a*) or in figure (*b*)? Explain. What would your response be if the flow direction is reversed?

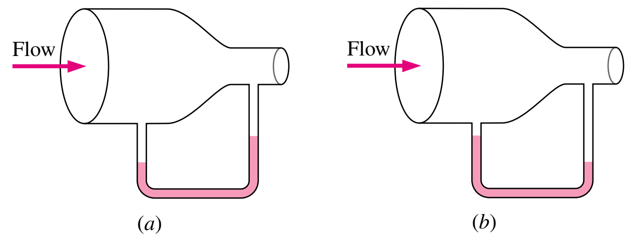

FIGURE P12–23C

12–24C The velocity of a fluid flowing in a pipe is to be measured by two different pitot-type mercury manometers shown in the figure. Would you expect both manometers to predict the same velocity for flowing water? If not, which would be more accurate? Explain. What would your response be if air were flowing in the pipe instead of water?

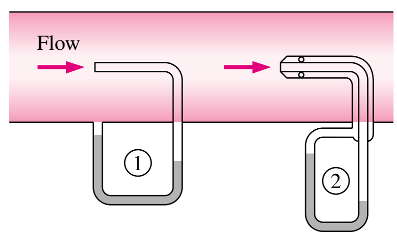

FIGURE P12–24

12–25 In cold climates, the water pipes may freeze and burst if proper precautions are not taken. In such an occurrence, the exposed part of a pipe on the ground ruptures, and water shoots up to 34 m. Estimate the gage pressure of water in the pipe. State your assumptions and discuss if the actual pressure is more or less than the value you predicted.

12–26 A pitot tube is used to measure the velocity of an aircraft flying at 3000 m. If the differential pressure reading is 3 kPa, determine the velocity of the aircraft.

12–27 While traveling on a dirt road, the bottom of a car hits a sharp rock and a small hole develops at the bottom of its gas tank. If the height of the gasoline in the tank is 30 cm, determine the initial velocity of the gasoline at the hole. Discuss how the velocity will change with time and how the flow will be affected if the lid of the tank is closed tightly. *Answer:* 2.43 m/s

12–28E The drinking water needs of an office are met by large water bottles. One end of a 0.25-in-diameter plastic hose is inserted into the bottle placed on a high stand, while the other end with an on/off valve is maintained 2 ft below the bottom of the bottle. If the water level in the bottle is 1.5 ft when it is full, determine how long it will take at the minimum to fill an 8-oz glass (□ 0.00835 ft^3) (*a*) when the bottle is first opened and (*b*) when the bottle is almost empty.

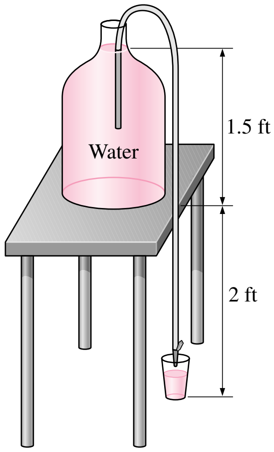

FIGURE P12–28E

12–29 A piezometer and a pitot tube are tapped into a 3-cm-diameter horizontal water pipe, and the height of the water columns are measured to be 20 cm in the piezometer and 35 cm in the pitot tube (both measured from the top surface of the pipe). Determine the velocity at the center of the pipe.

12–30 The diameter of a cylindrical water tank is D_o and its height is H. The tank is filled with water, which is open to the atmosphere. An orifice of diameter D_o with a smooth entrance (i.e., no losses) is open at the bottom. Develop a relation for the time required for the tank (*a*) to empty halfway and (*b*) to empty completely.

12–31 A pressurized tank of water has a 10-cm-diameter orifice at the bottom, where water discharges to the atmosphere. The water level is 3 m above the outlet. The tank air pressure above the water level is 300 kPa (absolute) while the atmospheric pressure is 100 kPa. Neglecting frictional effects, determine the initial discharge rate of water from the tank.
Answer: 0.168 m^3/s

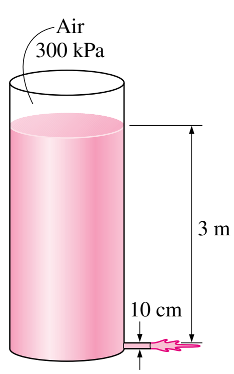

FIGURE P12–31

12–32 Reconsider Prob. 12–31. Using EES (or other) software, investigate the effect of water height in the tank on the discharge velocity. Let the water height vary from 0 to 5 m in increments of 0.5 m. Tabulate and plot the results.

12–33E A siphon pumps water from a large reservoir to a lower tank that is initially empty. The tank also has a rounded orifice 15 ft below the reservoir surface where the water leaves the tank. Both the siphon and the orifice diameters are 2 in. Ignoring frictional losses, determine to what height the water will rise in the tank at equilibrium.

12–34 Water enters a tank of diameter D_T steadily at a mass flow rate of $\dot{m}_{in}$. An orifice at the bottom with diameter D_o allows water to escape. The orifice has a rounded entrance, so the frictional losses are negligible. If the tank is initially empty, (*a*) determine the maximum height that the water will reach in the tank and (*b*) obtain a relation for water height z as a function of time.

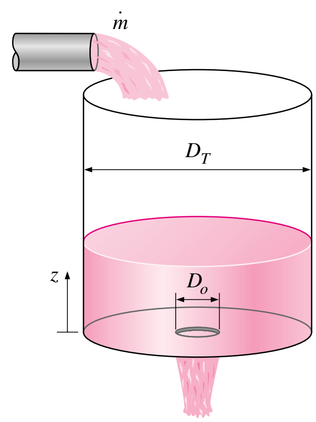

FIGURE P12–34

12–35E Water flows through a horizontal pipe at a rate of 1 gal/s. The pipe consists of two sections of diameters 4 in and 2 in with a smooth reducing section. The pressure difference between the two pipe sections is measured by a mercury manometer. Neglecting frictional effects, determine the differential height of mercury between the two pipe sections.
Answer: 0.52 in

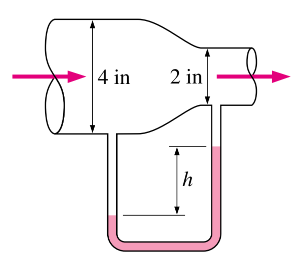

FIGURE P12–35E

12–36 An airplane is flying at an altitude of 12,000 m. Determine the gage pressure at the stagnation point on the nose of the plane if the speed of the plane is 200 km/h. How would you solve this problem if the speed were 1050 km/h? Explain.

12–37 The air velocity in the duct of a heating system is to be measured by a pitot tube inserted into the duct parallel to flow. If the differential height between the water columns connected to the two outlets of the pitot tube is 2.4 cm, determine (*a*) the flow velocity and (*b*) the pressure rise at the tip of the pitot tube. The air temperature and pressure in the duct are 45°C and 98 kPa.

12–38 The water in a 10-m-diameter, 2-m-high above-the-ground swimming pool is to be emptied by unplugging a 3-cm-diameter, 25-m-long horizontal pipe attached to the bottom of the pool. Determine the maximum discharge rate of water through the pipe. Also, explain why the actual flow rate will be less.

12–39 Reconsider Prob. 12–38. Determine how long it will take to empty the swimming pool completely.
Answer: 19.7 h

12–40 Reconsider Prob. 12–39. Using EES (or other) software, investigate the effect of the discharge pipe diameter on the time required to empty the pool completely. Let the diameter vary from 1 to 10 cm in increments of 1 cm. Tabulate and plot the results.

12–41 Air at 110 kPa and 50°C flows upward through a 6-cm-diameter inclined duct at a rate of 45 L/s. The duct diameter is then reduced to 4 cm through a reducer. The pressure change across the reducer is measured by a water manometer. The elevation difference between the two points on the pipe where the two arms of the manometer are attached is 0.20 m. Determine the differential height between fluid levels of the two arms of the manometer.

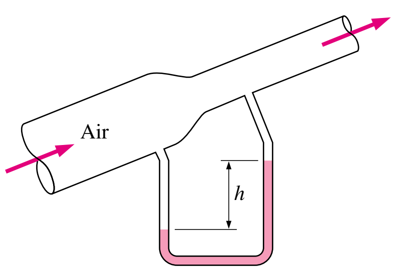

FIGURE P12–41

12–42E Air is flowing through a venturi meter whose diameter is 2.6 in at the entrance part (location 1) and 1.8 in at the throat (location 2). The gage pressure is measured to be 12.2 psia at the entrance and 11.8 psia at the throat. Neglecting frictional effects, show that the volume flow rate can be expressed as

$$\dot{V} = A_2\sqrt{\frac{2(P_1 - P_2)}{\rho(1 - A_2^2/A_1^2)}}$$

and determine the flow rate of air. Take the air density to be 0.075 lbm/ft^3.

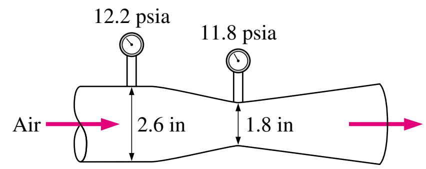

FIGURE P12–42E

12–43 The water pressure in the mains of a city at a particular location is 400 kPa gage. Determine if this main can serve water to neighborhoods that are 50 m above this location.

12–44 A handheld bicycle pump can be used as an atomizer to generate a fine mist of paint or pesticide by forcing air at a high velocity through a small hole and placing a short tube between the liquid reservoir and the high-speed air jet whose low pressure drives the liquid up through the tube. In such an atomizer, the hole diameter is 0.3 cm, the vertical distance between the liquid level in the tube and the hole is 10 cm, and the bore (diameter) and the stroke of the air pump are 5 cm and 20 cm, respectively. If the atmospheric conditions are 20°C and 95 kPa, determine the minimum speed that the piston must be moved in the cylinder during pumping to initiate the atomizing effect. The liquid reservoir is open to the atmosphere.

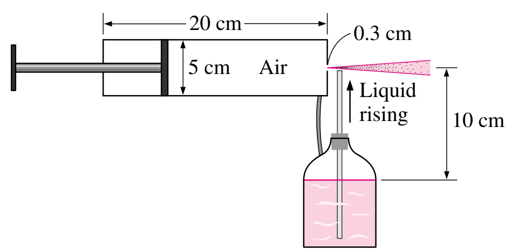

FIGURE P12–44

12–45 The water level in a tank is 20 m above the ground. A hose is connected to the bottom of the tank, and the nozzle at the end of the hose is pointed straight up. The tank cover is airtight, and the air pressure above the water surface is 2 atm

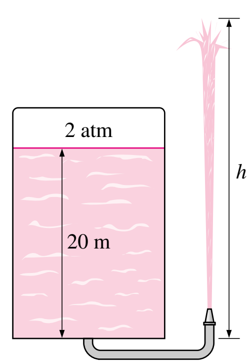

FIGURE P12–45

gage. The system is at sea level. Determine the maximum height to which the water stream could rise. *Answer:* 40.7 m

12–46 A pitot tube connected to a water manometer is used to measure the velocity of air. If the deflection (the vertical distance between the fluid levels in the two arms) is 7.3 cm, determine the air velocity. Take the density of air to be 1.25 kg/m^3.

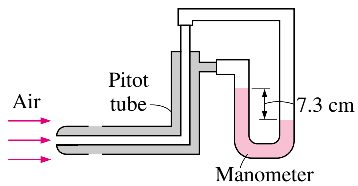

FIGURE P12–46

12–47E The air velocity in a duct is measured by a pitot tube connected to a differential pressure gage. If the air is at 13.4 psia absolute and 70°F and the reading of the differential pressure gage is 0.15 psi, determine the air velocity.
Answer: 142.7 ft/s

12–48 In a hydroelectric power plant, water enters the turbine nozzles at 700 kPa absolute with a low velocity. If the exit pressure is the atmospheric pressure of 100 kPa, determine the maximum velocity to which water can be accelerated by the nozzles before striking the turbine blades.

Energy Equation

12–49C Consider the steady adiabatic flow of an incompressible fluid. Can the temperature of the fluid decrease during flow? Explain.

12–50C Consider the steady adiabatic flow of an incompressible fluid. If the temperature of the fluid remains constant during flow, is it accurate to say that the frictional effects are negligible?

12–51C What is head loss? How is it related to the mechanical energy loss?

12–52C What is pump head? How is it related to the power input to the pump?

12–53C What is the kinetic energy correction factor? Is it significant?

12–54C The water level in a tank is 20 m above the ground. A hose is connected to the bottom of the tank, and the nozzle at the end of the hose is pointed straight up. The water stream from the nozzle is observed to rise 25 m above the ground. Explain what may cause the water from the hose to rise above the tank level.

12–55 Underground water is to be pumped by a 70 percent efficient 3-kW submerged pump to a pool whose free surface is 30 m above the underground water level. The diameter of the pipe is 5 cm on the intake side and 7 cm on the discharge side. Determine (*a*) the maximum flow rate of water and (*b*) the pressure difference across the pump. Assume the elevation difference between the pump inlet and the outlet to be negligible.

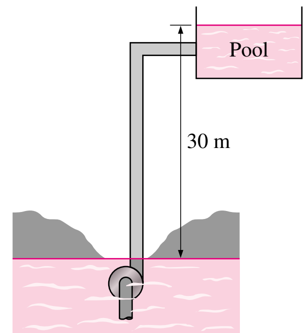

FIGURE P12–55

12–56 Reconsider Prob. 12–55. Determine the flow rate of water and the pressure difference across the pump if the head loss of the piping system is 5 m.

12–57E In a hydroelectric power plant, water flows from an elevation of 240 ft to a turbine, where electric power is generated. For an overall turbine-generator efficiency of 83 percent, determine the minimum flow rate required to generate 100 kW of electricity. *Answer:* 370 lbm/s

12–58E Reconsider Prob. 12–57E. Determine the flow rate of water if the head loss of the piping system between the free surfaces of the source and the sink is 36 ft.

12–59 A fan is to be selected to ventilate a bathroom whose dimensions are $2 \text{ m} \times 3 \text{ m} \times 3 \text{ m}$. The air velocity is not to exceed 8 m/s to minimize vibration and noise. The combined efficiency of the fan-motor unit to be used can be taken to be 50 percent. If the fan is to replace the entire volume of air in 10 min, determine (*a*) the wattage of the fan-motor unit to be purchased, (*b*) the diameter of the fan casing, and (*c*) the pressure difference across the fan. Take the air density to be 1.25 kg/m^3.

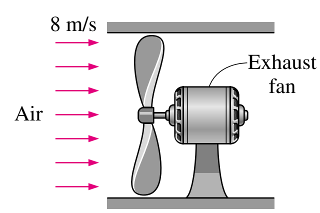

FIGURE P12–59

12–60 Water is being pumped from a large lake to a reservoir 25 m above at a rate of 25 L/s by a 10-kW (shaft) pump. If the head loss of the piping system is 7 m, determine the mechanical efficiency of the pump. *Answer:* 78.5%

12–61 Reconsider Prob. 12–60. Using EES (or other) software, investigate the effect of head loss on mechanical efficiency of the pump. Let the head loss vary from 0 to 15 m in increments of 1 m. Plot the results, and discuss them.

12–62 A 7-hp (shaft) pump is used to raise water to a 15-m higher elevation. If the mechanical efficiency of the pump is 82 percent, determine the maximum volume flow rate of water.

12–63 Water flows at a rate of 0.035 m^3/s in a horizontal pipe whose diameter is reduced from 15 cm to 8 cm by a reducer. If the pressure at the centerline is measured to be 470 kPa and 440 kPa before and after the reducer, respectively, determine the head loss in the reducer. *Answer:* 0.79 m

12–64 The water level in a tank is 20 m above the ground. A hose is connected to the bottom of the tank, and the nozzle at the end of the hose is pointed straight up. The tank is at sea level, and the water surface is open to the atmosphere. In the line leading from the tank to the nozzle is a pump, which increases the pressure of water. If the water jet rises to a height of 27 m from the ground, determine the minimum pressure rise supplied by the pump to the water line.

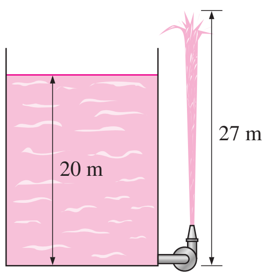

FIGURE P12–64

12–65 A hydraulic turbine has 85 m of head available at a flow rate of 0.25 m^3/s, and its overall turbine-generator efficiency is 78 percent. Determine the electric power output of this turbine.

12–66 The demand for electric power is usually much higher during the day than it is at night, and utility companies often sell power at night at much lower prices to encourage consumers to use the available power generation capacity and to avoid building new expensive power plants that will be used only a short time during peak periods. Utilities are also willing to purchase power produced during the day from private parties at a high price.

Suppose a utility company is selling electric power for $0.03/kWh at night and is willing to pay $0.08/kWh for power produced during the day. To take advantage of this opportunity, an entrepreneur is considering building a large reservoir 40 m above the lake level, pumping water from the lake to the reservoir at night using cheap power, and letting the water flow from the reservoir back to the lake during the day, producing power as the pump-motor operates as a turbine-generator during reverse flow. Preliminary analysis shows that a water flow rate of 2 m^3/s can be used in either direction, and the head loss of the piping system is 4 m. The combined pump-motor and turbine-generator efficiencies are expected to be 75 percent each. Assuming the system operates for 10 h each in the pump and turbine modes during a typical day, determine the potential revenue this pump-turbine system can generate per year.

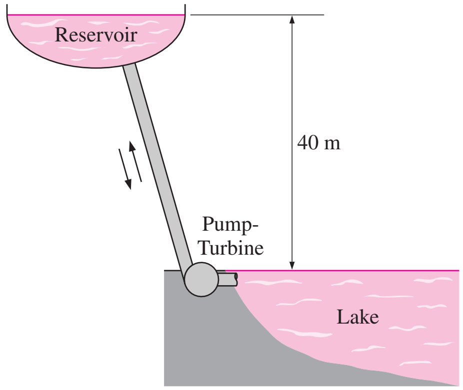

FIGURE P12–66

12–67 Water flows at a rate of 20 L/s through a horizontal pipe whose diameter is constant at 3 cm. The pressure drop across a valve in the pipe is measured to be 2 kPa. Determine the head loss of the valve, and the useful pumping power needed to overcome the resulting pressure drop.
Answers: 0.204 m, 40 W

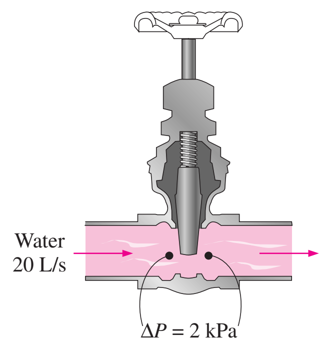

FIGURE P12–67

12–68E The water level in a tank is 66 ft above the ground. A hose is connected to the bottom of the tank, and the nozzle at the end of the hose is pointed straight up. The tank cover is airtight, but the pressure over the water surface is unknown. Determine the minimum tank air pressure (gage) that will cause a water stream from the nozzle to rise 90 ft from the ground.

12–69 A large tank is initially filled with water 2 m above the center of a sharp-edged 10-cm-diameter orifice. The tank water surface is open to the atmosphere, and the orifice drains to the atmosphere. If the total head loss in the system is 0.3 m, determine the initial discharge velocity of water from the tank.

12–70 Water enters a hydraulic turbine through a 30-cm-diameter pipe at a rate of 0.6 m^3/s and exits through a 25-cm-diameter pipe. The pressure drop in the turbine is measured by a mercury manometer to be 1.2 m. For a combined turbine-generator efficiency of 83 percent, determine the net electric power output.

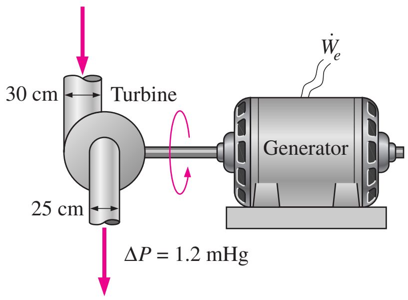

FIGURE P12–70

12–71 The velocity profile for turbulent flow in a circular pipe is usually expressed as $u(r) = u_{max}(1 - r/R)^{1/n}$ where $n = 7$. Determine the kinetic energy correction factor for this flow. *Answer:* 1.06

12–72 An oil pump is drawing 35 kW of electric power while pumping oil with $\rho = 860$ kg/m^3 at a rate of 0.1 m^3/s. The inlet and outlet diameters of the pipe are 8 cm and 12 cm, respectively. If the pressure rise of oil in the pump is measured to be 400 kPa and the motor efficiency is 90 percent, determine the mechanical efficiency of the pump.

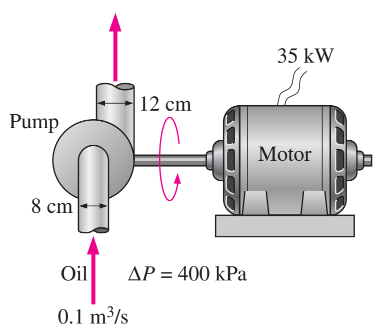

FIGURE P12–72

12–73E A 73-percent efficient 12-hp pump is pumping water from a lake to a nearby pool at a rate of 1.2 ft^3/s through a constant-diameter pipe. The free surface of the pool is 35 ft above that of the lake. Determine the head loss of the piping system, in ft, and the mechanical power used to overcome it.

12–74 A fireboat is to fight fires at coastal areas by drawing seawater with a density of 1030 kg/m^3 through a 20-cm-diameter pipe at a rate of 0.1 m^3/s and discharging it through a hose nozzle with an exit diameter of 5 cm. The total head loss of the system is 3 m, and the position of the nozzle is 4 m above sea level. For a pump efficiency of 70 percent, determine the required shaft power input to the pump and the water discharge velocity. *Answers:* 200 kW, 50.9 m/s

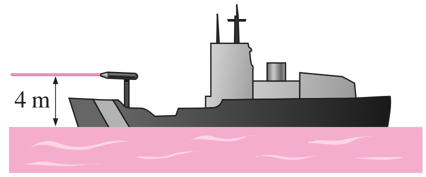

FIGURE P12–74

Review Problems

12–75 A pressurized 2-m-diameter tank of water has a 10-cm-diameter orifice at the bottom, where water discharges to the atmosphere. The water level initially is 3 m above the outlet. The tank air pressure above the water level is maintained at 450 kPa absolute and the atmospheric pressure is 100 kPa. Neglecting frictional effects, determine (*a*) how long it will take for half of the water in the tank to be discharged and (*b*) the water level in the tank after 10 s.

12–76 Air flows through a pipe at a rate of 200 L/s. The pipe consists of two sections of diameters 20 cm and 10 cm with a smooth reducing section that connects them. The pressure difference between the two pipe sections is measured by a water manometer. Neglecting frictional effects, determine the differential height of water between the two pipe sections. Take the air density to be 1.20 kg/m^3. *Answer:* 3.7 cm

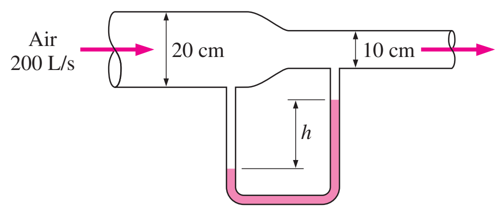

FIGURE P12–76

12–77 Air at 100 kPa and 25°C flows in a horizontal duct of variable cross section. The water column in the manometer that measures the difference between two sections has a vertical displacement of 8 cm. If the velocity in the first section is low and the friction is negligible, determine the velocity at the second section. Also, if the manometer reading has a possible error of ±2 mm, conduct an error analysis to estimate the range of validity for the velocity found.

12–78 A very large tank contains air at 102 kPa at a location where the atmospheric air is at 100 kPa and 20°C. Now a 2-cm-diameter tap is opened. Determine the maximum flow rate of air through the hole. What would your response be if air is discharged through a 2-m-long, 4-cm-diameter tube with a 2-cm-diameter nozzle? Would you solve the problem the same way if the pressure in the storage tank were 300 kPa?

12–79 Water is flowing through a venturi meter whose diameter is 7 cm at the entrance part and 4 cm at the throat. The pressure is measured to be 430 kPa at the entrance and 120 kPa at the throat. Neglecting frictional effects, determine the flow rate of water. *Answer:* 0.538 m^3/s

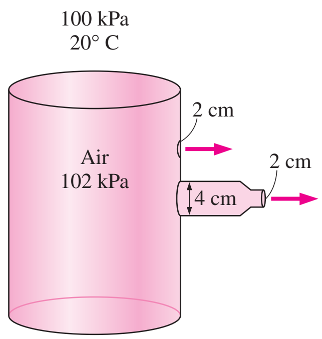

FIGURE P12–78

12–80E The water level in a tank is 80 ft above the ground. A hose is connected to the bottom of the tank, and the nozzle at the end of the hose is pointed straight up. The tank is at sea level, and the water surface is open to the atmosphere. In the line leading from the tank to the nozzle is a pump, which increases the water pressure by 10 psia. Determine the maximum height to which the water stream could rise.

12–81 A wind tunnel draws atmospheric air at 20°C and 101.3 kPa by a large fan located near the exit of the tunnel. If the air velocity in the tunnel is 80 m/s, determine the pressure in the tunnel.

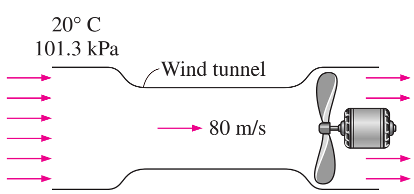

FIGURE P12–81

12–82 Water flows at a rate of 0.025 m^3/s in a horizontal pipe whose diameter increases from 6 cm to 11 cm by an enlargement section. If the head loss across the enlargement section is 0.45 m, determine the pressure change.

12–83 A 2-m-high large tank is initially filled with water. The tank water surface is open to the atmosphere, and a sharp-edged 10-cm-diameter orifice at the bottom drains to the atmosphere through a horizontal 100-m-long pipe. If the total head loss of the system is determined to be 1.5 m, determine the initial velocity of the water from the tank.
Answer: 3.13 m/s

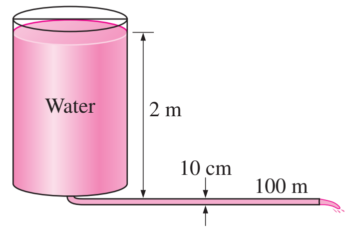

FIGURE P12–83

12–84 Reconsider Prob. 12–83. Using EES (or other) software, investigate the effect of the tank height on the initial discharge velocity of water from the completely filled tank. Let the tank height vary from 2 to 15 m in increments of 1 m, and assume the head loss to remain constant. Tabulate and plot the results.

12–85 Reconsider Prob. 12–83. In order to drain the tank faster, a pump is installed near the tank exit. Determine the pump head input necessary to establish an average water velocity of 6 m/s when the tank is full.

Design and Essay Problems

12–86 Your company is setting up an experiment that involves the measurement of airflow rate in a duct, and you are to come up with proper instrumentation. Research the available techniques and devices for airflow rate measurement, discuss the advantages and disadvantages of each technique, and make a recommendation.

12–87 Computer-aided designs, the use of better materials, and better manufacturing techniques have resulted in a tremendous increase in the efficiency of pumps, turbines, and electric motors. Contact several pump, turbine, and motor manufacturers and obtain information about the efficiency of their products. In general, how does efficiency vary with rated power of these devices?

12–88 Using a handheld bicycle pump to generate an air jet, a soda can as the water reservoir, and a straw as the tube, design and build an atomizer. Study the effects of various parameters such as the tube length, the diameter of the exit hole, and the pumping speed on performance.

12–89 Using a flexible drinking straw and a ruler, explain how you would measure the water flow velocity in a river.